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DIRECTION FINDING FOR DISTRIBUTED SIGNALS

by

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ABSTRACT

In this report, the problem of estimating the directions of arrival (DOA) of distributed signals is examined. A mathematical model is first established by making some reasonable assumptions. The correlation matrix of a distributed signal is then derived. Examples of signals with different distributions are presented. The important properties of the correlation matrix are studied revealing that even the matrix is of full rank (being equal to the number of sensors) which renders the conventional high resolution array processing methods inapplicable in this case, the dimensionality of the signal subspace can be approximated to a number usually much smaller than the number of sensors. From this observation and utilizing the orthogonality of the signal and noise subspaces, two groups of algorithms (DISPARE and VEC-MUSIC) have been developed. Several computer simulation examples are carried out to examine the performance of the algorithms under various environments. It is observed that, in general, the algorithm DISPARE 1 may not be robust in the sense that it may yield unacceptable results under some particular conditions. On the other hand, DISPARE 2 generally yields the least estimation error, being slightly better than VEC-MUSIC. However, the VEC-MUSIC algorithms, in general, have the additional flexibility of choosing the noise subspace (called the *extended pure* and *extended quasi* noise subspaces) as well as the ability of distinguishing a distributed source from a point source.

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I INTRODUCTION

I.1 Sensor Array Processing

Array processing, or more accurately, *sensor array signal processing*, is the processing of the output signals of an array of sensors located at different points in space in a wavefield. The purpose of array processing is to extract useful information from the received signals such as the number and the location of the signal sources, the propagation velocity of the waves, as well as the spectral properties of the signals. Array processing techniques have been employed in various areas in which very different wave phenomena occur. These include seismic exploration, passive sonar, radar, radio astronomy, as well as radio communications.

Common to all these applications, there are, in general, two essential purposes in array processing:-
(i) To determine the number of signal sources (decision),
(ii) To estimate the locations of these sources (estimation),
(iii) To extract a desired signal from interference.

The discussions in this report are focused on the second aspect. Furthermore, we only consider the case of one-dimensional arrays in this report although the discussion can be readily extended to arrays of higher dimensions.

The traditional way of tackling the problems in array processing is based on procedures employing the fast Fourier transform (FFT) [1-6]. This approach is computationally efficient and produces reasonable results for a large class of signal environments. It is also robust in the sense that it does not depend on a signal model. In spite of these advantages, there are several inherent performance limitations of the FFT methods. The most important limitation is that of spatial resolution, i.e., the ability to distinguish the spatial response of two or more signal sources. The spatial resolution of an FFT array processor is roughly inversely proportional to the size of the ar-

ray. A second limitation is due to the implicit weighting of the sensor outputs (array shading) that occurs when processing with the FFT. Array shading manifests itself as "leakage" in the spatial domain, i.e., energy in the main lobe of a spatial response "spills" into the sidelobes, obscuring and distorting other spatial responses that are present. In fact, weak signal spatial responses can be masked by higher sidelobes from stronger spatial responses. Careful application of tapered sensor apertures can reduce the sidelobe leakage, but always at the expense of reduced resolution.

In an attempt to overcome the inherent limitations of the FFT approach, many alternative array processing procedures, generally known as *high-resolution methods*, have been proposed in the past several years. Both problems of decision and estimation have been addressed and have usually been treated separately [7-14] [15-38]. These methods generally start with a signal and noise model, and, based on the covariance matrix of the received data, the data space is partitioned into the *signal subspace* and the *noise subspace*. Various methods are then applied to estimate the parameters concerning the number and the directions of arrival (DOA) of the signal sources in these subspaces.

In establishing these high resolution array processing methods, there are several assumptions made on the signal and noise model. Different methods may have additional assumptions, but the ones most commonly made are:–

1.1. The signal and noise processes are stationary over the observation interval and are uncorrelated with each other.
1.2. The number of sensors in the array is greater than the number of signal sources.
1.3. The sensors are properly calibrated at the time of the measurement.
1.4. The background noise is isotropic (spatially white), or, in the case of spatially non-white noise, the covariance matrix of the noise is known.
1.5. Signals are propagated from distinct point sources.

I.2 Estimation of the DOA of Point sources in Spatially White Noise

We first review the problem of estimating the DOA of signals in a spatially white noise environment. The signal model we consider here consists of an M-dimensional complex data vector $\boldsymbol{x}(n) \in \mathcal{C}^M$ which represents the data received by an array of M sensors at the nth snapshot. The data vector is composed of plane-wave incident narrowband signals each of angular frequency ω_0 from K distinct point sources embedded in Gaussian noise. The received data vector at the nth snapshot can be written as

$$\boldsymbol{x}(n) = \boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{s}(n) + \boldsymbol{\nu}(n), \qquad n = 1, \cdots, N \tag{I.1}$$

where N is the total number of snapshots observed. $\boldsymbol{s}(n) \in \mathcal{C}^K$ is the unknown K-dimensional complex vector whose elements represent the phasors of the signals at the nth snapshot. $\boldsymbol{\theta} \in \mathcal{R}^K$ is the K-dimensional real vector the elements of which represent the DOA of the K plane-wave signals such that

$$\boldsymbol{\theta} = [\theta_1 \ \cdots \ \theta_K]^T \tag{I.2}$$

where ^{T} denotes the transpose of a vector or matrix. The vector $\boldsymbol{\theta}$ in Eq. (I.2) is the vector to be estimated. The matrix $\boldsymbol{D}(\boldsymbol{\theta}) \in \mathcal{C}^{M \times K}$ has the following structure

$$\boldsymbol{D}(\boldsymbol{\theta}) = [\boldsymbol{d}(\theta_1) \ \cdots \ \boldsymbol{d}(\theta_K)] \triangleq [\boldsymbol{d}_1 \ \cdots \ \boldsymbol{d}_K] \tag{I.3}$$

where $\boldsymbol{d}_k \triangleq \boldsymbol{d}(\theta_k) \in \mathcal{C}^M, \quad k = 1, \cdots, K$, is an M-dimensional complex vector usually called the *directional vector* or sometimes also the *steering vector* of the signals. The exact form of the steering vector depends on the array configuration. Most of the discussions in this report do not require a specific form of $\boldsymbol{d}(\theta_k)$, i.e., they are valid for any general array geometry. However, the uniform linear array, apart from being most commonly used, may also offer advantageous implementation efficiency of some algorithm. A uniform linear array is shown in Fig. I.1 where the array elements are uniformly spaced. For a velocity of propagation c, the distance between two sensors in a uniform linear array must be $L_0 \leq \pi c/\omega_0$ and the corresponding steering matrix is given by

$$\boldsymbol{d}(\theta_k) = [1 \quad e^{j\omega_0 L_0 \sin\theta_k / c} \quad \cdots \quad e^{j(M-1)\omega_0 L_0 \sin\theta_k / c}]^T \tag{I.4}$$

where $\theta_k \in (-\frac{\pi}{2}, \frac{\pi}{2})$ for all k.

The vectors $\boldsymbol{d}(\theta_k), k = 1, \cdots, K$, corresponding to K different values of θ_k are assumed to be linearly

independent. This implies that $M > K$, and

$$\text{rank}(\boldsymbol{D}) = K \tag{I.5}$$

The noise vector sequence $\{\boldsymbol{\nu}(n)\}$ is assumed to be zero mean circular Gaussian such that

$$\text{E}[\boldsymbol{\nu}(n_1)\boldsymbol{\nu}^\dagger(n_2)] = \sigma_\nu^2 I \delta_{n_1 n_2} \tag{I.6a}$$

and

$$\text{E}[\boldsymbol{\nu}(n_1)\boldsymbol{\nu}^T(n_2)] = \boldsymbol{O} \tag{I.6b}$$

where † denotes the conjugate transpose operation,

$$\delta_{n_1 n_2} = \begin{cases} 0 & n_1 \neq n_2 \\ 1 & n_1 = n_2 \end{cases}$$

and σ_ν^2 is the noise power. Furthermore, $\boldsymbol{\nu}(n_1)$ and $\boldsymbol{s}(n_2)$ are assumed to be uncorrelated for all n_1 and n_2. Thus, if we form the correlation matrix of the received data $\boldsymbol{x}(n)$, using the above assumptions on the noise vector, we have

$$\boldsymbol{R}_x \triangleq \text{E}[\boldsymbol{x}\boldsymbol{x}^\dagger] = \boldsymbol{D}\boldsymbol{R}_s\boldsymbol{D}^\dagger + \sigma_\nu^2 I \tag{I.7}$$

where

$$\boldsymbol{R}_s \triangleq \text{E}[\boldsymbol{s}\boldsymbol{s}^\dagger] \tag{I.8}$$

and is positive definite.

In practice, the true value of $\boldsymbol{R}_x$ is seldom known. Therefore, an estimate $\hat{\boldsymbol{R}}_x(N)$ is obtained by averaging the outer products of the data over the total number of snapshots N such that

$$\hat{\boldsymbol{R}}_x(N) = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{x}(n)\boldsymbol{x}^\dagger(n) \tag{I.9}$$

where $\hat{}$ denotes the estimated value of a quantity. In the ensuing materials, the symbol $\hat{}$ appearing over a variable shall indicate the estimate of the variable based on $\hat{\boldsymbol{R}}_x(N)$. It can be shown [11, 14] that under certain conditions the estimate $\hat{\boldsymbol{R}}_x(N)$ converges to $\boldsymbol{R}_x$ with probability 1 and at a rate proportional to $(\log\log N \,/\, N)^{1/2}$, i.e.,

$$\lim_{N\to\infty} \hat{\boldsymbol{R}}_x(N) \underset{a.s.}{=} \boldsymbol{R}_x \tag{I.10}$$

where " $\underset{a.s.}{=}$ " indicates that the equality holds with probability 1, or almost surely. In particular, if the signal s is Gaussian, then under some regular conditions, the convergence of $\hat{\boldsymbol{R}}_x$ to $\boldsymbol{R}_x$ is at the rate of $O(N^{-1/2})$ in probability [47, 71]. In the following text, the explicit dependence of $\hat{\boldsymbol{R}}_x$ on N will be omitted to maintain conciseness of the expression.

In the past decade or two, various high resolution methods of estimating the DOA of the signal sources based upon the above model have been developed. Most of these methods are based on the partition of the space of the observed data into the signal and the noise subspaces assuming that the number of signals K is known. Different techniques are then employed to locate the DOA of the signals using these estimated subspaces. A method has withstood the test of time and has remained popular among researchers in high resolution array processing:

The MUSIC Algorithm

The *Multiple Signal Classification* (MUSIC) algorithm [24] of DOA estimation in high resolution array processing assumes, in addition to Assumptions 1.1—1.5, that the K signal sources are incoherent with each other; and that the number K has been correctly determined.

We begin by examining the eigenvalues λ_m and the corresponding normalized eigenvectors $\boldsymbol{v}_m$, $m = 1, \cdots, M$, of $\boldsymbol{R}_x$. Since $\boldsymbol{R}_x$ is Hermitian and positive definite [70], its eigenvalues are real and positive, and the eigenvectors are orthonormal. Furthermore, $\boldsymbol{R}_x$ can be decomposed such that [67, 68, 70]

$$\boldsymbol{R}_x = \sum_{m=1}^{M} \lambda_m \boldsymbol{v}_m \boldsymbol{v}_m^\dagger \tag{I.11}$$

where $\lambda_1 > \lambda_2 > \cdots > \lambda_K > \lambda_{K+1} = \cdots = \lambda_M = \sigma_\nu^2$, and

$$\boldsymbol{v}_m^\dagger \boldsymbol{v}_n = 0 \quad \text{for } m \neq n \tag{I.12}$$

From the definition of eigenvalues and eigenvectors, we can write

$$(\boldsymbol{R}_x - \sigma_\nu^2 \boldsymbol{I})\boldsymbol{v}_m = \boldsymbol{O} \qquad m = K+1, \cdots, M \tag{I.13}$$

Using Eq. (I.7), Eq. (I.13) can be rewritten as

$$\boldsymbol{D}\boldsymbol{R}_s\boldsymbol{D}^\dagger\boldsymbol{v}_m = \boldsymbol{O} \quad m = K+1, \cdots, M \tag{I.14}$$

Since the signals are incoherent, $\boldsymbol{R}_s$ is full rank. Also, since the signal sources are from K distinct directions, $\boldsymbol{D}$ is of full column rank. Thus, we can conclude that

$$\boldsymbol{D}^\dagger\boldsymbol{v}_m = \boldsymbol{O} \quad m = K+1, \cdots, M \tag{I.15}$$

Eq. (I.15) tells us that each of the K directional vectors $\boldsymbol{d}_k$ is orthogonal to the eigenvectors $\{\boldsymbol{v}_m\}$, $m = K+1, \cdots, M$, of the matrix $\boldsymbol{R}_x$. But from Eq. (I.12), we see that the eigenvectors $\{\boldsymbol{v}_k\}$, $k = 1, \cdots, K$, span the orthogonal complement of the space spanned by $\{\boldsymbol{v}_m,\ m = K+1, \cdots, M\}$. Hence we can deduce that

$$\text{span}\{\boldsymbol{d}_1, \cdots, \boldsymbol{d}_K\} = \text{span}\{\boldsymbol{v}_1, \cdots, \boldsymbol{v}_K\} \tag{I.16}$$

Therefore, we can partition the eigenvectors of $\boldsymbol{R}_x$ into two groups such that

$$\boldsymbol{V}_s \triangleq [\boldsymbol{v}_1\ \boldsymbol{v}_2\ \cdots\ \boldsymbol{v}_K] \tag{I.17a}$$

and

$$\boldsymbol{V}_\nu \triangleq [\boldsymbol{v}_{K+1}\ \boldsymbol{v}_{K+2}\ \cdots\ \boldsymbol{v}_M] \tag{I.17b}$$

From Eqs. (I.15) and (I.16), we see that the eigenvectors of $\boldsymbol{R}_x$ span two disjoint subspaces. The one spanned by the eigenvectors corresponding to the K largest eigenvalues of $\boldsymbol{R}_x$ is called the *signal subspace* because it is also spanned by the directional vectors of the signals. The one spanned by the eigenvectors corresponding to the $(M-K)$ smallest eigenvalues of $\boldsymbol{R}_x$ is called the *noise subspace* because of its association with the noise power σ_ν^2. The principle of the MUSIC algorithm is to determine the DOA of the signals by searching for the directional vectors $\boldsymbol{d}_k$, $k = 1, \cdots, K$, that are orthogonal to the noise subspace.

As mentioned before, in practice we can only estimate $\boldsymbol{R}_x$ by $\hat{\boldsymbol{R}}_x$ given by Eq. (2.9). Since $\hat{\boldsymbol{R}}_x$ is still Hermitian and positive definite, its eigenvalues, denoted by $\hat{\lambda}_m$, $m = 1, \cdots, M$, are still positive, and the associated eigenvectors, denoted by $\{\hat{\boldsymbol{v}}_m\}$, $m = 1, \cdots, M$, are still mutually orthonormal. However, the smallest $(M-K)$ eigenvalues $\hat{\lambda}_{K+1}, \cdots, \hat{\lambda}_M$ will not be equal any

more, and the estimated noise subspace spanned by $\{\hat{v}_m\}$, $m = K+1, \cdots, M$, will no longer be orthogonal to the true signal subspace spanned by $\{d_k\}$, $k = 1, \cdots, K$. The best that we can do is to search for the directional vectors which are closest to being orthogonal to the noise subspace. Accordingly, we form an *orthogonal projector* [40, 41] such that

$$\hat{P}_\nu = \hat{V}_\nu (\hat{V}_\nu^\dagger \hat{V}_\nu)^{-1} \hat{V}_\nu^\dagger = \hat{V}_\nu \hat{V}_\nu^\dagger \tag{I.18}$$

where $\hat{V}_\nu = [\hat{v}_{K+1}, \cdots, \hat{v}_M]$. The second equality in Eq. (2.18) is due to the orthonormality of the eigenvectors. We note that

$$\hat{P}_\nu^\dagger = \hat{P}_\nu \tag{I.19a}$$

and

$$\hat{P}_\nu^\dagger \hat{P}_\nu = \hat{P}_\nu \tag{I.19b}$$

We can now use $\hat{P}_\nu$ to project the general directional vector $d(\theta)$ onto the estimated noise subspace spanned by the column vectors of $\hat{V}_\nu$. The values of $\hat{\theta}_k$, $k = 1, \cdots, K$, which give the K highest peaks of the quantity $1/\| \hat{P}_\nu d(\theta) \|^2$ are the estimated DOA of the K signal sources. In other words, the estimates of DOA for the MUSIC algorithm can be written as

$$\hat{\theta}_k = \arg\min_{\theta_k} \{\|\hat{P}_\nu d(\theta_k)\|^2\} = \arg\min_{\theta_k} \{d^\dagger(\theta_k) \hat{P}_\nu d(\theta_k)\} \tag{I.20}$$

where $\arg\min_{\theta_k}(\cdot)$ denotes the minimization of the bracketed quantity with respect to the argument θ_k.

Suggestions have been made to weight the projection $\hat{P}_\nu d(\theta_k)$ in different ways resulting in various criteria of *weighted MUSIC* algorithms [20, 27]. The MUSIC method remains one of the most popular methods of DOA estimation in high resolution array processing because of its relative simplicity in computation as well as its relative robustness and efficiency [39-44, 46-48].

I.3 DOA Estimation in Non-Ideal Environments

Analyses and simulation tests on MUSIC and other high resolution methods have been carried out [39-48] and it has been shown that under the appropriate assumptions, these methods have

superior performance compared to the FFT approach in the cases involving multiple and closely spaced signal sources. However, when the environment departs from its ideal assumptions, the performance of these methods generally deteriorates.

The five assumptions generally made in high resolution array processing methods have different practical implications. The first assumption can usually be satisfied, at least to a close approximation, in practice. The second assumption does not represent a harsh practical restriction either. However Assumptions 1.3, 1.4 and 1.5 are not easily satisfied in practice.

Assumption 1.3 is not easy to satisfy because, in practice, calibrating an antenna array can be time consuming and expensive. In addition to the problem of initial calibration, there is the problem of maintaining proper calibration of the sensors. The factors contributing to the gradual changes in the response of the sensors and their associated circuits are many and varied: Changes in the environment around the array may affect the response (e.g. coupling) of the sensors, changes in the location of the sensors may introduce perturbation of the sensor positions, aging of components, moisture, and, temperature variation all contribute to changes in the response of the sensors. It is impossible in many practical situations to maintain array calibration to the accuracy required for the proper operation of these high-resolution methods. Analyses and simulation tests [49-51] show that improperly calibrated sensor arrays may lead to unacceptable estimation results. Self-calibration algorithms [52-56] for high resolution array processing have been proposed to alleviate the problems introduced by imprecise array calibration. These methods generally use the received signals to fine-tune the array calibration and are usually based on the simultaneous estimation of the DOA of the signal sources and of the unknown array parameters. However, estimation of both signal and array parameters is a) computationally complex, b) hampered by potential identifiable problems.

Assumption 1.4, on the other hand, is often invalid in practice. Background noise, or *ambient noise*, refers to the noise that remains after all easily identifiable signal sources are eliminated. The sources of ambient noise can be both natural and man-made, with different sources exhibiting different directional and spectral characteristics. In a passive sonar system for instance [57, 58], the

natural sources of noise include seismic disturbances, agitation of the sea surfaces by wind, thermal activity of the water molecules, as well as biological sources such as snapping shrimps, dolphins, and other fishes and ocean mammals. The principal man-made component of ambient noise, on the other hand, is the sound generated by distant shipping. Hence, the ambient noise characteristics for such cases are highly dependent on geographic location, the transmission medium, season of the year, and weather, and are, in general, unknown. Thus, Assumption 1.4 is often an over-simplified model which may not represent the true spatial characteristics of the noise.

The problem of DOA estimation in unknown correlated noise has been studied by various authors [59-65]. The most complete and effective methods, however, have been proposed by Wong, Wu, and Stoica [72-74] in which two arrays of sensors are used and the *canonical correlation decomposition* is applied to the two sets of data obtained. This leads to very accurate estimates of the signal and noise subspace from which various high resolution estimation methods can be established.

Assumption 1.5 is, at best, an approximation of the practical environment. The "size" of a source depends on its distance from the array of sensors. In addition, signal sources, in practice, may often be transmitted by reflection and the reflective medium may often be dispersed and thus violating the "point source" assumption. In the case of communication in the Arctic environment for instance, the transmission of radio waves is often performed by ionospheric scattering. Fig.I.2 is a simplified diagram illustrating such a situation. The signal having undergone reflection through a scattering volume reaches the receiver and would appear to be from a distributed source. The receiver, in this application, has to determine the DOA of this distributed signal. A second example arises in the case of a radar system performing low angle tracking. This case is illustrated in Fig.I.3 in which the aircraft, if it is sufficient far away, can be approximately modelled as a point source. However, the diffused components caused by the reflection from a rough sea surface would appear as a distributed source. Here, the objective may be to estimate the DOA of a point source in the presence of distributed sources. Since the existing methods of high resolution array processing rely on the assumption that the signals are from point sources, the application of these methods to such problems will show grave deterioration in performance. It is on this particular problem that our

attention is focused in this report. We approach the problem by developing a signal model for a distributed source relative to tha array of sensors. From this model, the spatial correlation matrix of the data received by the sensors is established. The important properties of this correlation matrix are analysed and examined. Based upon these properties, several methods of estimating the DOA of distributed and point sources are developed. Computer simulations are then carried out to test and compared the effectiveness of these methods. Some observations are drawn from the simulation results and future work on the subject is discussed.

I.4 Notation and Glossary

The following is a list of the notational conventions for vectors and matrices used in this report:

Vectors and matrices in a real space $\mathcal{R}$ or a complex space $\mathcal{C}$ are usually denoted by bold, lower cases and by bold, upper cases respectively. In addition, for a complex matrix of dimension $M \times N$, we have

$$
\begin{aligned}
\boldsymbol{A}^T &= \text{the transpose of } \boldsymbol{A}, \\
\boldsymbol{A}^* &= \text{the complex conjugate of } \boldsymbol{A}, \\
\boldsymbol{A}^\dagger &= \text{the conjugate transpose of } \boldsymbol{A}, \\
\boldsymbol{A}^{-1} &= \text{the inverse of } \boldsymbol{A} \in \mathcal{C}^{M \times M}, \\
\boldsymbol{A}^\# &= \text{generalized inverse of } \boldsymbol{A}, \\
\| \boldsymbol{A} \| &= \text{Frobenius norm of } \boldsymbol{A}, \\
\mathrm{Re}[\boldsymbol{A}] &= \text{the real part of } \boldsymbol{A}, \\
\mathrm{Im}[\boldsymbol{A}] &= \text{the imaginary part of } \boldsymbol{A}, \\
\mathrm{tr}[\boldsymbol{A}] &= \text{the trace of } \boldsymbol{A} \in \mathcal{C}^{M \times M}, \\
\det[\boldsymbol{A}] &= \text{the determinant of } \boldsymbol{A} \in \mathcal{C}^{M \times M}, \\
[\boldsymbol{A}]_{mn} &= \text{the } mn\text{th element of } \boldsymbol{A},
\end{aligned}
$$

$$
\begin{aligned}
(\boldsymbol{A})_{\cdot j} &= \text{the } j\text{th column vector of } \boldsymbol{A}, \\
(\boldsymbol{A})_{i\cdot} &= \text{the } i\text{th row vector of } \boldsymbol{A}, \\
\text{vec}(\boldsymbol{A}) &= [a_{11} \cdots a_{M1}\, a_{12} \cdots a_{M2} \cdots a_{1N} \cdots a_{MN}]^T, \\
\boldsymbol{A} \otimes \boldsymbol{B} &= \text{the Kronecker product of } \boldsymbol{A} \text{ and } \boldsymbol{B} \\
&= \begin{bmatrix} a_{11}\boldsymbol{B} & \cdots & a_{1N}\boldsymbol{B} \\ \cdot & \cdots & \cdot \\ a_{M1}\boldsymbol{B} & \cdots & a_{MN}\boldsymbol{B} \end{bmatrix}, \\
\boldsymbol{A} \odot \boldsymbol{B} &= \text{the Hadamard product of two matrices of same dimension} \\
&= \begin{bmatrix} a_{11}b_{11} & \cdots & a_{1N}b_{1N} \\ \cdot & \cdots & \cdot \\ a_{M1}b_{M1} & \cdots & a_{MN}b_{MN} \end{bmatrix}.
\end{aligned}
$$

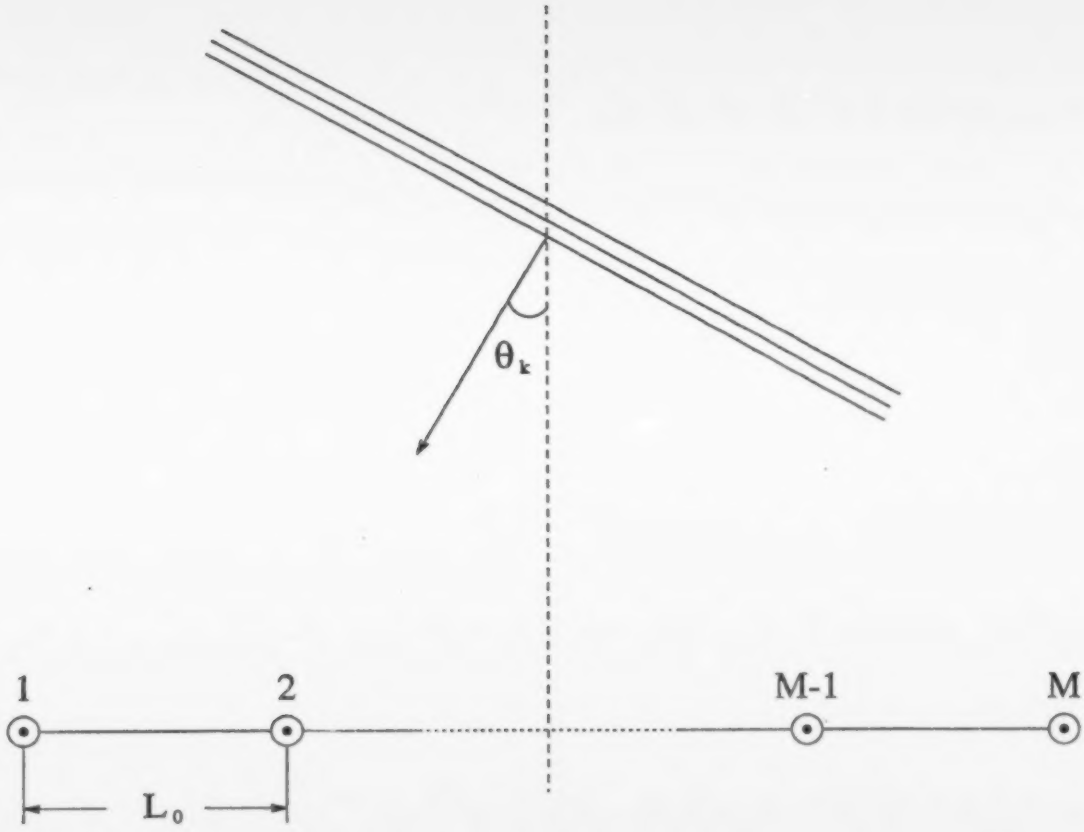


Figure I.1: An incident point signal and a linear uniform sensor array

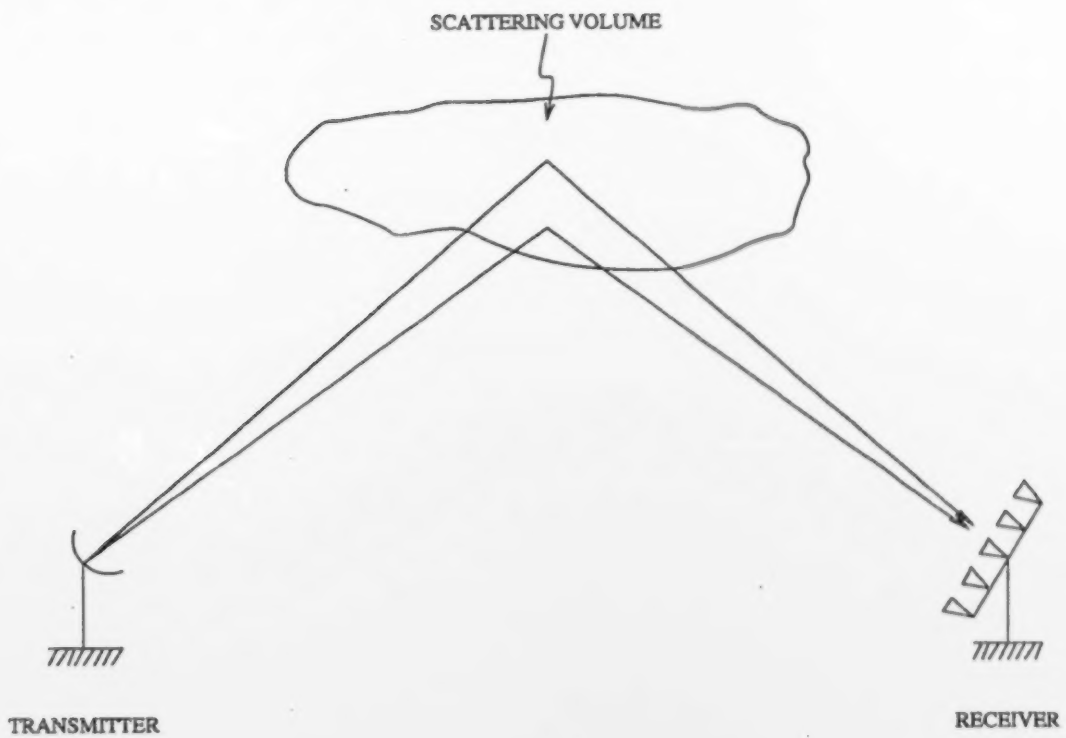


Figure I.2: Signals transmitted by scattering propagation

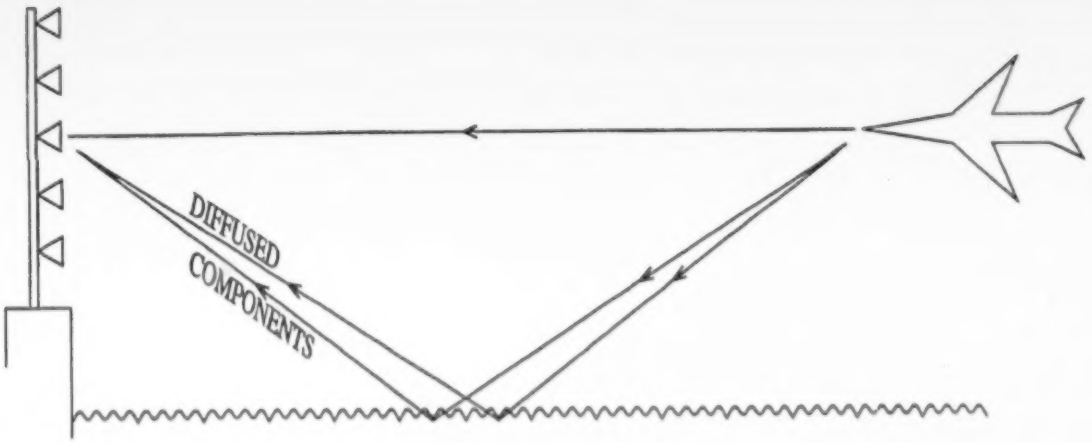


Figure I.3: Low-angle tracking environments

II Signal Model for a Distributed Source in Spatially White Noise Environment

Consider an array of M sensors monitoring a spatially distributed source originating from a far distance, therefore the signal waves incident upon the array can be regarded as plane waves. For the formulation of the problem, a uniform linear array is being considered here, because uniform linear arrays are most commonly used and the signal model for a uniform linear array can be simplified by confining our attention on a plane in which the array and the distributed signal lie. Furthermore, most of the time, DOA estimation algorithms have many computational advantages for uniform linear arrays.

We use a coordinate system as shown in Fig.II.1 to depict the scenario. M omidirectional sensors are uniformly aligned on the u_1-axis with the first sensor at the origin. The signal from one elemental source of the scattering volume, which subtends a solid angle $d\Omega$, is coming from the direction defined by elevation angle θ and azimuth angle ψ.

Given a narrow band signal

$$s(t) = ae^{j\omega_0 t} \tag{II.1}$$

where a is the random complex envelope of the signal and ω_0 is the centre frequency.

The signal density from the elemental source at the direction (θ, ψ) received by sensor m_1 with reference to sensor 1 is given by

$$s_{m_1}(\theta, \psi, t) = a(\theta, \psi)e^{j[\omega_0 - \tau_{m_1}(\theta,\psi)]} \tag{II.2}$$

where $\tau_{m_1}(\theta, \psi)$ is the time delay between the wave front propagation from sensor 1 to sensor m_1. For the uniform linear array shown in Fig.II.1, it is equal to

$$\tau_{m_1}(\theta, \psi) = L_0(m_1 - 1)\sin\theta/c \tag{II.3}$$

where c is the velocity of the wave front and L_0 is the distance between two adjacent sensors. From

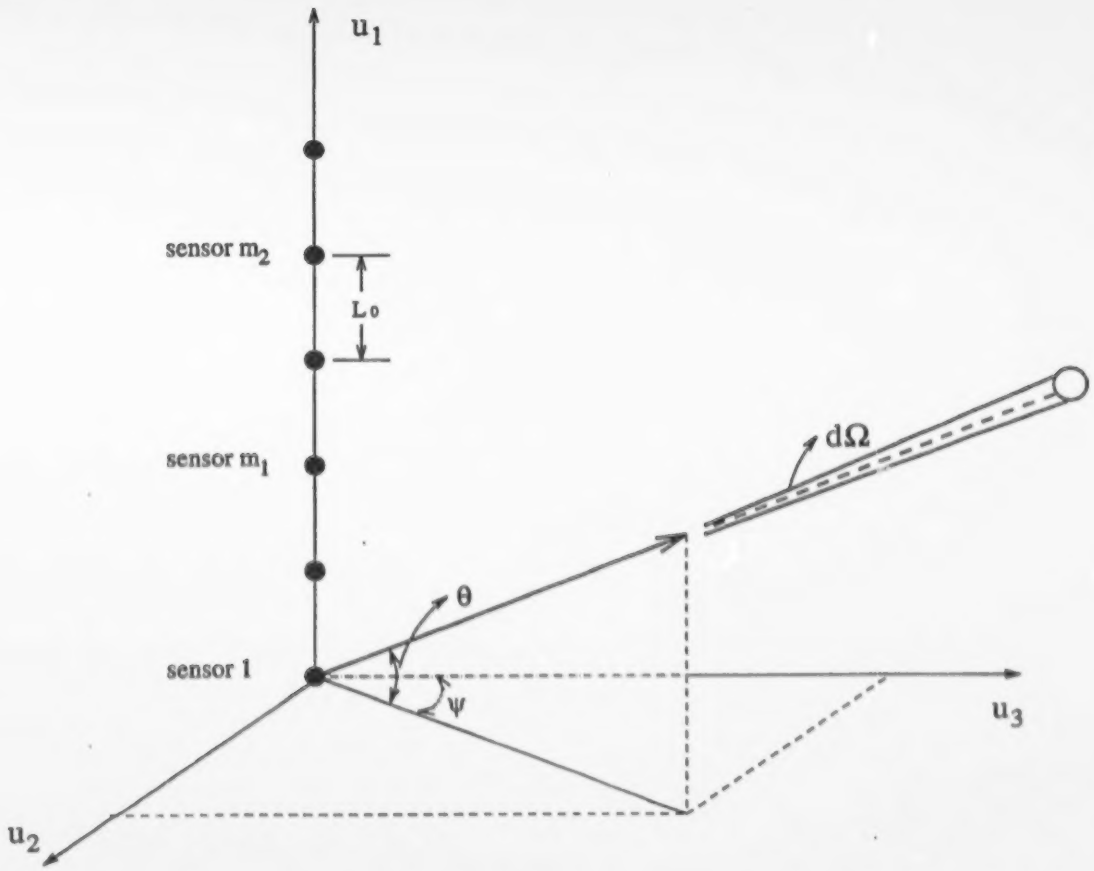


Figure II.1: The coordinate system

Fig.II.1, we can see that, for the different sensors, the propagation time delay only depends on the angle θ, different angle ψ will not introduce time delay among sensors.

The total signal received by sensor m_1 can be obtained by integrating the signal density with respect to the entire scattering volume, i.e.,

$$s_{m_1}(t) = \int_{4\pi} a(\theta, \psi) e^{j\omega_0[t-\tau_{m_1}(\theta,\psi)]} d\Omega \tag{II.4}$$

Similarly, we can obtain the signal received by sensor m_2.

The spatial correlation between signals received by sensors m_1 and m_2 is given by

$$\begin{aligned} & R_{s_{m_1} s_{m_2}}(m_1, m_2) \\ = \; & \mathrm{E}[s_{m_1}(t) s^*_{m_2}(t)] \\ = \; & \int_{4\pi}\int_{4\pi} \mathrm{E}\{a(\theta_i, \psi_i) a^*(\theta_j, \psi_j)\} e^{j\omega_0[\tau_{m_2}(\theta_j,\psi_j) - \tau_{m_1}(\theta_i,\psi_i)]} d\Omega_i d\Omega_j \end{aligned} \tag{II.5}$$

We assume that the signal from different elemental sources are uncorrelated, i.e.,

$$\mathrm{E}\{a(\theta_i, \psi_i) a^*(\theta_j, \psi_j)\} = \mathrm{E}\{|a(\theta_i, \psi_i)|^2\} \delta_{ij} \tag{II.6}$$

where δ_{ij} is the Kronecker delta. This assumption usually holds in the transmission of the radio waves by inospheric scattering as well as in the reception of reflected signals from rough surfaces. Therefore, using Eq.(II.6), the spatial correlation of Eq.(II.5) can be written as

$$R_{s_{m_1} s_{m_2}}(m_1, m_2) = \int_{4\pi} \mathrm{E}\{|a(\theta_i, \psi_i)|^2\} e^{j\omega_0[\tau_{m_2}(\theta_i,\psi_i) - \tau_{m_1}(\theta_i,\psi_i)]} d\Omega_i \tag{II.7}$$

For the uniform linear array in Fig.II.1, using Eq.(II.3), Eq.(II.7) can be simplified to

$$R_{s_{m_1} s_{m_2}}(m_1, m_2) = \int_{4\pi} \mathrm{E}\{|a(\theta_i, \psi_i)|^2\} e^{j\omega_0 (m_2 - m_1) L_0 \sin\theta_i / c} d\Omega_i \tag{II.8}$$

The elemental solid angle $d\Omega_i$ is defined by the area of subtending elemental surface divided by radius square. By using the spherical coordinate system (r, θ, ψ), as seen in Fig.II.2, we have

$$d\Omega_i = \frac{dA_i}{r^2} = \frac{r\cos\theta_i d\psi_i r d\theta_i}{r^2} = d\sin\theta_i d\psi_i \tag{II.9}$$

Therefore, by substituting Eq.(II.9) into Eq.(II.8), the spatial correlation function can be written as

$$\begin{aligned} & R_{s_{m_1} s_{m_2}}(m_1, m_2) \\ = \; & \int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi} \mathrm{E}\{|a(\theta_i, \psi_i)|^2\} d\psi_i \, e^{j(m_2 - m_1)\omega_0 L_0 \sin\theta_i / c} d\sin\theta_i \\ = \; & \int_{-\pi}^{\pi} |\, p(\phi)\,|^2 \, e^{j(m_2 - m_1)\phi} d\phi \end{aligned} \tag{II.10}$$

where we define

$$\phi = \omega_0 L_0 \sin\theta_i / c$$

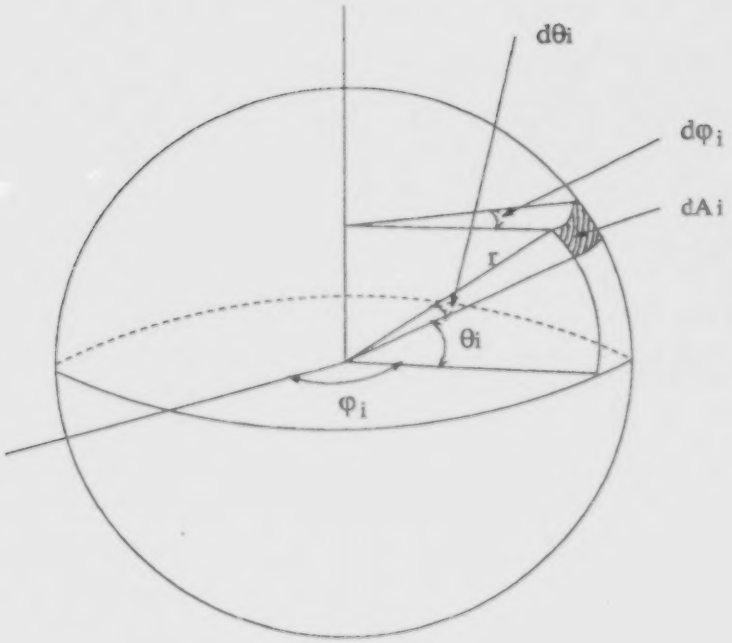


Figure II.2: The spherical coordinate system (r, θ, ψ)

and

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{E}[\mid a_o(\theta_i, \psi_i) \mid^2 d\psi_i = \mid p(\phi) \mid^2 \qquad \text{(II.11)}$$

The signal spectrum density $|p(\phi)|^2$ in Eq.(II.11) defines the distribution in the source-array plane of the scattering volume from which the distributed source is generated. If the distribution $|p(\phi)|^2$ can be defined by two parameters $\bar{\phi}$ and σ_ϕ which are, respectively, the mean angle of arrival and the "spread" of the signal, then the correlation matrix can be reduced to one which depends only on these two parameters.

For a linear array, we can confine our attention to a one-dimension DOA estimation. For instance, the array shown in Fig.II.1 is only used to estimate the elevation angle. For an array with arbitary geometry, except when the distributed signal is flat and coplanar with the array, DOA estimation usually involves two angle, such as the simultaneous estimation of the angles of elevation and azimuth.

We can formulate the signal received by a linear array from a distributed source in another way by conceiving the signal density spectrum comprised of a collection of discrete sources and develop a discrete model which may be convenient for computation.

In the discrete model, we conceive these discrete signal sources to be co-planar with the array of sensors. There are L discrete sources altogether, the lth one being from an electrical angle ϕ_l with reference to the array. Thus, the signal received by the array is given by

$$\boldsymbol{s}(n) = \sum_{l=1}^{L} \boldsymbol{d}(\phi_l) p_l a_l(n) = \boldsymbol{D}\boldsymbol{P} \begin{bmatrix} a_1(n) \\ \vdots \\ a_L(n) \end{bmatrix} \tag{II.12}$$

where $\boldsymbol{D} = [\boldsymbol{d}(\phi_1) \cdots \boldsymbol{d}(\phi_L)]$, $\boldsymbol{d}(\phi_l)$ is the array steering vector at angle ϕ_l; $\boldsymbol{P} = \mathrm{diag}(p_1 \cdots p_L)$, denoting the strength of each discrete signal source and $\{a_l(n)\}$ are normalized uncorrelated Gaussian random variables with zero mean and unit variance.

Hence, the correlation matrix $\boldsymbol{R}_{ss}$ of the received signal is

$$\begin{aligned} \boldsymbol{R}_{ss} &= \mathrm{E}\{\boldsymbol{s}(n)\boldsymbol{s}^{\dagger}(n)\} \\ &= \sum_{l=1}^{L} \boldsymbol{d}(\phi_l)\boldsymbol{d}^{\dagger}(\phi_l) p_l^2 \end{aligned} \tag{II.13}$$

If we choose large enough number of discrete sources, i.e., we represent the signal spectrum by a large number of discrete point sources separated by small intervals $\triangle\phi$, then we can approximate the "power" p_l^2 of the lth signal as $|p(\phi)|^2 \triangle\phi$, so that Eq.(II.13) can be rewritten as

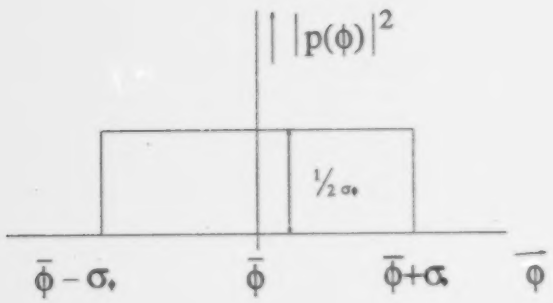
$$\begin{aligned} \boldsymbol{R}_{ss} &= \lim_{\triangle\phi \to 0} \sum_{l=1}^{L} \boldsymbol{d}(\phi_l)\boldsymbol{d}^{\dagger}(\phi_l)|p(\phi)|^2 \triangle\phi \\ &= \int_{-\pi}^{\pi} \boldsymbol{d}(\phi)\boldsymbol{d}^{\dagger}(\phi)|p(\phi)|^2 d\phi \end{aligned} \tag{II.14}$$

which is the same as the continuous model in Eq.(II.10). In our study here, we also normalize the total power from the entire distributed source so that

$$\int_{-\pi}^{\pi} |p(\phi)|^2 d\phi = 1 \tag{II.15}$$

The following are three examples of signal correlation matrix of different distributed sources.

Example 1: Uniform Power Density



Suppose that the signal power density is uniformly distributed, i.e.,

$$|\,p(\phi)\,|^2 = \begin{cases} \frac{1}{2\sigma_\phi} & |\,\phi-\bar{\phi}\,| < \sigma_\phi \\ 0 & \text{elsewhere} \end{cases} \tag{II.16}$$

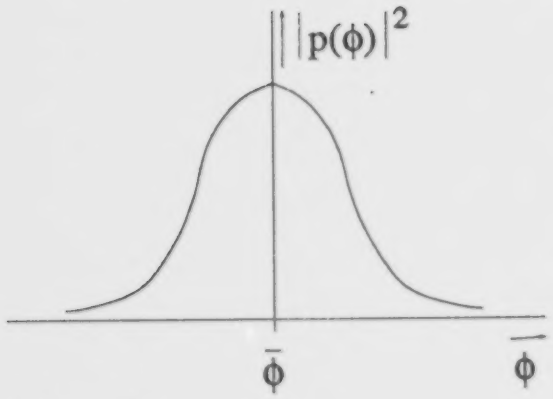
Note that the total area under the power density spectrum is unity due to normalization. For a uniform linear array, using Eq.(II.14), the mnth entry of the correlation matrix is given by

$$r_{ss\,mn} = \int_{-\pi}^{\pi} e^{j(m-n)\phi}|p(\phi)|^2 d\phi = \frac{\sin(m-n)\sigma_\phi}{(m-n)\sigma_\phi} e^{j(m-n)\bar{\phi}} \tag{II.17}$$

Hence, the correlation matrix of the signal received by the array is

$$\boldsymbol{R}_{ss} = \begin{bmatrix} 1 & \frac{\sin\sigma_\phi}{\sigma_\phi}e^{-j\bar{\phi}} & \cdots & \frac{\sin(M-1)\sigma_\phi}{(M-1)\sigma_\phi}e^{-j(M-1)\bar{\phi}} \\ \frac{\sin\sigma_\phi}{\sigma_\phi}e^{j\bar{\phi}} & 1 & \cdots & \frac{\sin(M-2)\sigma_\phi}{(M-2)\sigma_\phi}e^{-j(M-2)\bar{\phi}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\sin(M-1)\sigma_\phi}{(M-1)\sigma_\phi}e^{j(M-1)\bar{\phi}} & . & \cdots & 1 \end{bmatrix} \tag{II.18}$$

Example 2: Gaussian Power Density



Given the power density of the signal is

$$| p(\phi) |^2 = \frac{1}{\sqrt{2\pi\sigma_\phi^2}} e^{-\frac{(\phi-\bar{\phi})^2}{2\sigma_\phi^2}} \tag{II.19}$$

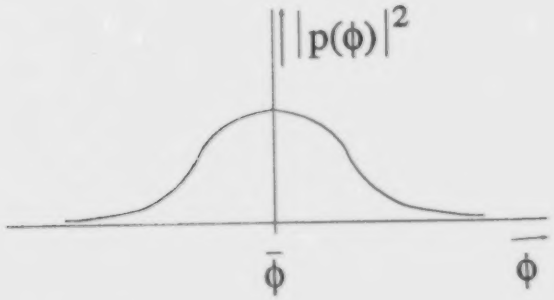
where we assume that the spread width $\sigma_\phi \ll \pi$. Again, by Eq.(II.14), the mnth entry of the correlation matrix $\boldsymbol{R}_{ss}$ is given by

$$r_{ss\,mn} = \int_{-\pi}^{\pi} e^{j(m-n)\phi} |p(\phi)|^2 d\phi = e^{-\frac{(m-n)^2\sigma_\phi^2}{2}} e^{j(m-n)\bar{\phi}} \tag{II.20}$$

Hence, the signal spatial correlation matrix for a uniform linear array is given as

$$\boldsymbol{R}_{ss} = \begin{bmatrix} 1 & e^{-\frac{\sigma_\phi^2}{2}} e^{-j\bar{\phi}} & \cdots & e^{-\frac{(M-1)^2\sigma_\phi^2}{2}} e^{-j(M-1)\bar{\phi}} \\ e^{-\frac{\sigma_\phi^2}{2}} e^{j\bar{\phi}} & 1 & \cdots & e^{-\frac{(M-2)^2\sigma_\phi^2}{2}} e^{-j(M-2)\bar{\phi}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-\frac{(M-1)^2\sigma_\phi^2}{2}} e^{j(M-1)\bar{\phi}} & e^{-\frac{(M-2)^2\sigma_\phi^2}{2}} e^{j(M-2)\bar{\phi}} & \cdots & 1 \end{bmatrix}$$

Example 3: "First-order Butterworth" Distribution



If the signal power density is given by

$$| p(\phi) |^2 = \frac{\sigma_\phi}{\sigma_\phi^2 + (\phi - \bar{\phi})^2} \tag{II.21}$$

where σ_ϕ is the $-3dB$ "width" of the spectrum of $| p(\phi) |^2$ and we assume $\sigma_\phi \ll \pi$. Using Eq.(II.14), the mnth entry of $\boldsymbol{R}_{ss}$ is given by

$$r_{ss\,mn} = \int_{-\pi}^{\pi} e^{j(m-n)\phi} |p(\phi)|^2 d\phi = e^{-(m-n)\sigma_\phi} e^{j(m-n)\bar{\phi}} \tag{II.22}$$

Hence, the correlation matrix of the signal with unity power is

$$\boldsymbol{R}_{ss} = \begin{bmatrix} 1 & z^* & z^{*2} & \cdots & z^{*(M-1)} \\ z & 1 & z^* & \cdots & z^{*(M-2)} \\ z^2 & z & 1 & \cdots & z^{*(M-3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z^{M-1} & z^{M-2} & z^{M-3} & \cdots & 1 \end{bmatrix} \tag{II.23}$$

where $z = e^{-\sigma_\phi} e^{j\bar{\phi}}$

From above three examples, we can see that for a distributed signal with symmetric power density about mean DOA $\bar{\phi}$ each entry of the signal correlation matrix has both the modulus part which is dependent on the spread σ_ϕ and argument part which is dependent on the mean DOA. For the case of unsymmetric spectrum, we can similarly separate each element of the correlation matrix into two parts one that depends on the spread only, the other that depends on the mean DOA only. However, the spread-dependent part will not be real in this case.

Under the assumption that this distributed signal source is corrupted by spatially uncorrelated ambient noise, the correlation matrix of the array output data $\boldsymbol{x}(n)$ is

$$R_{\boldsymbol{x}} = R_{\boldsymbol{ss}} + \sigma_{\nu}^2 I \tag{II.24}$$

where σ_{ν}^2 is the ambient noise power.

If there are K uncorrelated distributed signal source arriving at the array, again accompanied by white ambient noise which is uncorrelated with signals, the correlation matrix of the array output data $\boldsymbol{x}(n)$ is given by

$$R_{\boldsymbol{x}} = \sum_{k=1}^{K} R_{\boldsymbol{ss}k} + \sigma_{\nu}^2 I \tag{II.25}$$

III The Properties of the Correlation Matrix R_{ss}

As we have mentioned in Section II, if a distributed signal power density is symmetric to the mean DOA, then the correlation matrix $\boldsymbol{R_{ss}}$ is generally of the following form

$$\boldsymbol{R_{ss}} = \begin{bmatrix} r_{11} & r_{12}e^{-j\bar{\phi}} & \cdots & r_{1M}e^{-j(M-1)\bar{\phi}} \\ r_{21}e^{j\bar{\phi}} & r_{22} & \cdots & r_{2M}e^{-j(M-2)\bar{\phi}} \\ \cdot & \cdot & \cdots & \cdot \\ r_{M1}e^{j(M-1)\bar{\phi}} & r_{M2}e^{j(M-2)\bar{\phi}} & \cdots & r_{MM} \end{bmatrix} = \boldsymbol{ERE}^{\dagger} \qquad \text{(III.1)}$$

where

$$\boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1M} \\ r_{21} & r_{22} & \cdots & r_{2M} \\ \cdot & \cdot & \cdots & \cdot \\ r_{M1} & r_{M2} & \cdots & r_{MM} \end{bmatrix} \qquad \text{(III.2)}$$

and

$$\boldsymbol{E} = \begin{bmatrix} 1 & & & \boldsymbol{O} \\ & e^{j\bar{\phi}} & & \\ & & \ddots & \\ \boldsymbol{O} & & & e^{j(M-1)\bar{\phi}} \end{bmatrix} \qquad \text{(III.3)}$$

with r_{mn} being real quantities such that $r_{mn} = r_{nm}$ and only depend on the spread, and $\bar{\phi}$ being the mean DOA. If the signal power density is not symmetric to the mean DOA, then r_{mn} are generally complex quantities. We note that, in general, $\boldsymbol{R}$ is of full rank. Also, since $\boldsymbol{R}$ is Hermitian, its eigenvalues are real and non-negative, and its eigenvectors are mutually orthogonal.

Theorem III.1 *If matrix $\boldsymbol{R}$ has eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ with corresponding eigenvectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_M$, then $\boldsymbol{R_{ss}}$ has the same eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ with corresponding eigenvectors*

$$\boldsymbol{E}\boldsymbol{v}_m = \begin{bmatrix} 1 & & & \boldsymbol{O} \\ & e^{j\bar{\phi}} & & \\ & & \ddots & \\ \boldsymbol{O} & & & e^{j(M-1)\bar{\phi}} \end{bmatrix} \boldsymbol{v}_m, \quad m = 1, 2, \cdots, M \qquad \text{(III.4)}$$

Proof:

By eigen-decomposition, matrix $\boldsymbol{R}$ can be written as

$$\boldsymbol{R} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^{\dagger}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_M)$ is the eigenvalue matrix of $\boldsymbol{R}$ and $\boldsymbol{V} = [v_1, v_2, \cdots, v_M]$ is the eigenvector matrix of $\boldsymbol{R}$.

From Eq.(III.1), we know

$$\begin{aligned} \boldsymbol{R}_{ss} &= \boldsymbol{E}\boldsymbol{R}\boldsymbol{E}^{\dagger} \\ &= \boldsymbol{E}\boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^{\dagger}\boldsymbol{E}^{\dagger} \\ &= (\boldsymbol{E}\boldsymbol{V})\boldsymbol{\Lambda}(\boldsymbol{E}\boldsymbol{V})^{\dagger} \end{aligned}$$

where $\boldsymbol{E}$ is given by Eq.(III.3).

It is easy to verify that $\boldsymbol{E}\boldsymbol{V}$ is a unitary matrix. Therefore, $\boldsymbol{\Lambda}$ is the eigenvalue matrix and $\boldsymbol{E}\boldsymbol{V}$ is the eigenvector matrix of matrix $\boldsymbol{R}_{ss}$, i.e., $\boldsymbol{R}_{ss}$ has the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$ with corresponding eigenvectors $\boldsymbol{E}v_1$, $\boldsymbol{E}v_2$, $\cdots$ $\boldsymbol{E}v_M$. □

This theorem tells us that we only need to examine the real matrix $\boldsymbol{R}$ if we want to obtain the eigenvalues and eigenvectors of the signal correlation matrix $\boldsymbol{R}_{ss}$.

III.1 The Eigenvalues and Eigenvectors of a Uniformly Distributed Signal Correlation Matrix

From Example 1 in section II, we can see that for the case of uniformly distributed signal source, the $mnth$ element of the real matrix $\boldsymbol{R}$ is given by

$$r_{mn} = \frac{\sin(m-n)\sigma_\phi}{(m-n)\sigma_\phi} \quad m, n = 1, 2, \cdots, M \tag{III.5}$$

where $2\sigma_\phi$ is the total width of the distribution. Then the mth eigenvectors of the matrix $\boldsymbol{R}$ is formed by the elements of the *discrete prolate spheroidal sequences* $q_m^{(i)}(\sigma_\phi)$ (see Appendix A), and

the corresponding eigenvalue $\lambda_m(\sigma_\phi)$ is given by

$$\lambda_m(\sigma_\phi) = \frac{\pi \sum_{i=1}^{M} \mid q_m^{(i)}(\sigma_\phi) \mid^2}{\sigma_\phi \sum_{i=-\infty}^{\infty} \mid q_m^{(i)}(\sigma_\phi) \mid^2} \quad m = 1, \cdots, M \tag{III.6}$$

Table III.1 shows the numerical value of $\lambda_m(\sigma_\phi)$ for various width of σ_ϕ. We notice that after $m > m_0 = [\frac{\sigma_\phi M}{2}]$, where σ_ϕ is in radian, $\lambda_m(\sigma_\phi)$ is negligible. Here $[x]$ denotes the first integer greater than or equal to x. This means that most of the signal energy are concentrated on the first m_0 eigenvalues, therefore, the first m_0 eigenvectors corresponding to these eigenvalues span a subspace which is close to the real signal subspace with an approximation error of $\sum_{m=m_0+1}^{M} \lambda_m(\sigma_\phi)$ if measured by energy loss.

Table III.1 The eigenvalues of uniformly distributed signal

σ_ϕ	4.5°	6.7°	10°
λ_1	3.7367e-00	4.5941e-00	5.6473e-00
λ_2	4.4294e-01	9.5332e-01	2.4568e-00
λ_3	9.0845e-03	3.7703e-02	2.6517e-01
λ_4	6.3929e-05	4.8073e-04	8.2259e-03
λ_5	2.1313e-07	2.8643e-06	1.1242e-04
λ_6	3.6407e-10	8.7404e-09	7.7967e-07
λ_7	5.9799e-13	1.1789e-11	2.6663e-09
λ_8	2.8915e-13	1.4103e-12	1.2009e-12
$\vdots$	$\vdots$	$\vdots$	$\vdots$

III.2 The Eigenvalues and Eigenvectors of a Gaussian Distributed Signal Correlation Matrix

For the case of Gaussian distributed signal source, the elements of the real matrix $\boldsymbol{R}$ are given by

$$r_{mn} = e^{-(m-n)^2\sigma_\phi^2/2} \quad m, n = 1, 2, \cdots, M \tag{III.7}$$

where σ_ϕ is the width of the distribution.

The eigenvalues $\lambda_m(\sigma_\phi)$ and corresponding eigenvectors $\boldsymbol{\xi}_m(\sigma_\phi)$ can be obtained by solving the following equation by standard numerical subroutines,

$$\lambda_m(\sigma_\phi)\boldsymbol{\xi}_m(\sigma_\phi) = \boldsymbol{R}_{M,\sigma_\phi}\boldsymbol{\xi}_m(\sigma_\phi) \tag{III.8}$$

The eigenvalues are plotted in Fig.III.1 to Fig.III.3 for different σ_ϕ. We can see that $\lambda_1(\sigma_\phi) \geq \lambda_2(\sigma_\phi) \geq \cdots \geq \lambda_M(\sigma_\phi)$ and more and more eigenvalues $\lambda_m(\sigma_\phi)$ have significant value when M increases. We can also examine the numerical values of eigenvalues $\lambda_m(\sigma_\phi)$ from Table III.2, and we note that for a fixed value of σ_ϕ, $\lambda_m(\sigma_\phi)$ fall off to zero rapidly with increasing m once m has exceeds $m_0 = [\frac{\pi\sigma_\phi M}{2}]$, where again, $[\frac{\pi\sigma_\phi M}{2}]$ is the first integer greater than or equal to $\frac{\pi\sigma_\phi M}{2}$. Therefore, we can choose the first m_0 eigenvalues as the signal eigenvalues and corresponding eigenvectors as the basis spanning the approximated signal subspace.

Table III.2 The eigenvalues of Gaussian distributed signal

σ_ϕ	1.5°	2.2°	3.3°
λ_1	7.6969e-00	7.4822e-00	6.9524e-00
λ_2	2.9846e-01	5.0369e-01	9.8349e-01
λ_3	4.5987e-03	1.3898e-02	6.1805e-02
λ_4	4.1575e-05	2.2466e-04	2.2810e-03
λ_5	2.4320e-07	2.3514e-06	5.4649e-05
λ_6	9.2058e-10	1.5953e-08	8.5213e-07
λ_7	1.7032e-12	6.4096e-11	7.9857e-09
λ_8	3.7873e-13	5.5602e-13	3.3071e-11
⋮	⋮	⋮	⋮

III.3 The Dimensionality of Signal Subspace of a Distributed Source

The analysis in Section III.2 shows that although theoretically the matrix $\boldsymbol{R}$ is of full rank, for a practical range of spread σ_ϕ, almost all of the energy of the distributed signal concentrate on the first few eigenvalues. If we choose

$$m_0 = [\frac{\sigma_\phi M}{2}] \tag{III.9a}$$

for a uniformly distributed signal or

$$m_0 = [\frac{\pi\sigma_\phi M}{2}] \tag{III.9b}$$

for a Gaussian distributed signal as the dimensionality of the signal subspace, more than 95% of the signal energy will be retained in this subspace. Obviously, a higher m_0 will yield a more accurate approximation of dimensionality of signal subspace. Based on above analysis, we can establish an appropriate criterion to partition the signal subspace and noise subspace with small signal energy "leakage" into noise subspace such that an accurate DOA estimation can be guaranteed.

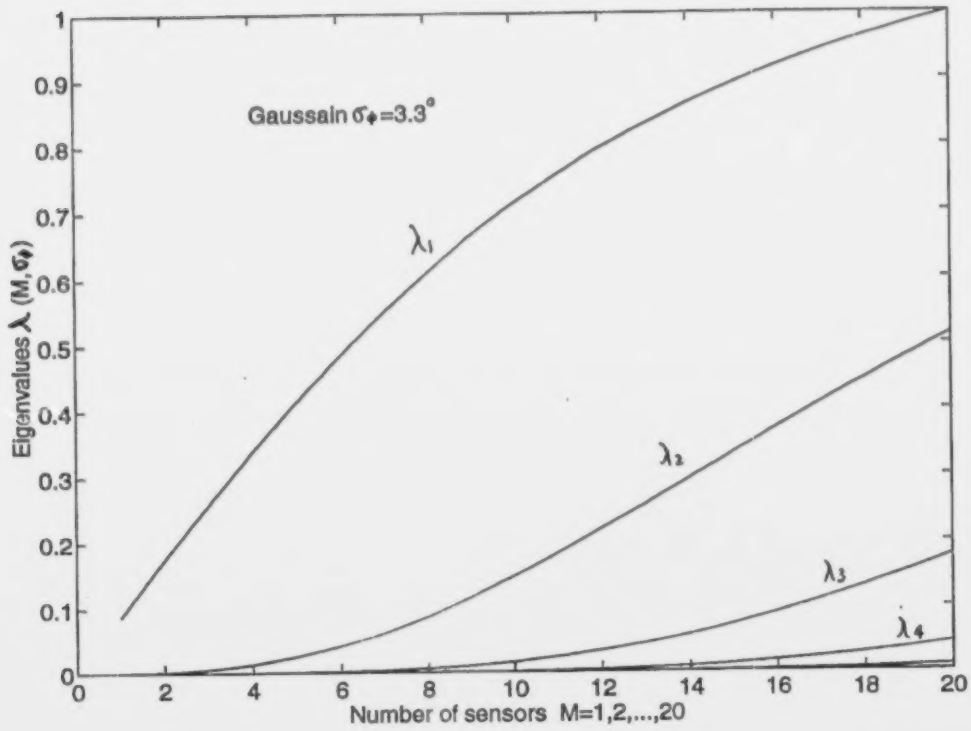


Figure III.1: The eigenvalues of Gaussian distributed signal with $\sigma_\phi = 3.3°$

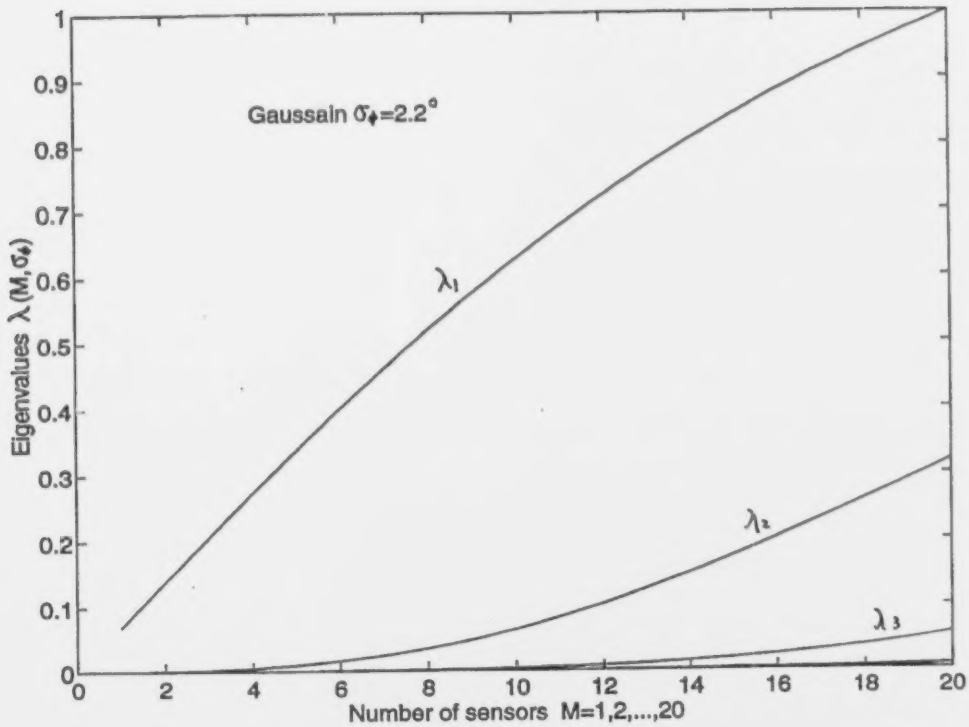


Figure III.2: The eigenvalues of Gaussian distributed signal with $\sigma_\phi = 2.2°$

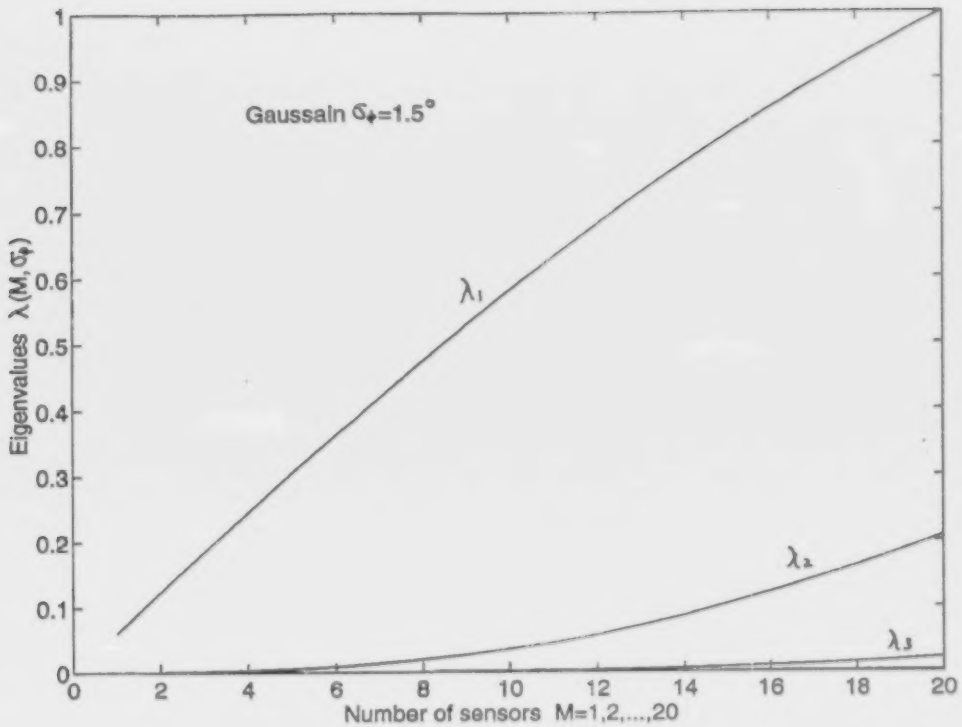


Figure III.3: The eigenvalues of Gaussian distributed signal with $\sigma_\phi = 1.5°$

IV Estimating the DOA of Distributed Signals

In this section, we develop the DOA estimation methods of a distributed signal source. The properties examined in Section III play an important part in developing methods of DOA estimation. The development of the algorithms is based on the signal model established in Section II. We also explore a DOA estimation method when a distributed signal and a point signal coexist. Common to all methods, we assume that the number of signals of point sources or distributed sources, K, has been correctly determined.

IV.1 Distributed Signal Parameter Estimator (DISPARE)

From the analysis of the correlation matrix $\boldsymbol{R}_{ss}$, We note that there are only a few eigenvalues which are of significance for a practical range of spread. The associated eigenvectors span the so-called signal subspace. From Theorem III.1, we know these eigenvectors bear the information of the mean DOA $\bar{\phi}$ and the spread σ_ϕ. The methods of DISPARE attempt to fit these eigenvectors of the distributed signal to the estimated noise eigenvectors by adjusting the parameters $\bar{\phi}$ and σ_ϕ until the two sets of eigenvectors are nearly mutually orthogonal. From this, we obtain the estimation of parameters $\bar{\phi}$ and σ_ϕ.

Consider the covariance matrix $\boldsymbol{R}_x$ of the received data when only one distributed signal is present under the assumption of spatially white ambient noise. From Eq.(II.24) we know that

$$\boldsymbol{R}_x = \boldsymbol{R}_{ss} + \sigma_\nu^2 I$$

where σ_ν^2 is the noise power. The matrix $\boldsymbol{R}_{ss}$ depends on the distribution geometry of the signal. Since the addition of $\sigma_\nu^2 I$ does not change the eigenvectors of a matrix, then both $\boldsymbol{R}_x$ and $\boldsymbol{R}_{ss}$ have the same eigenvectors, denoted by $\{v_m, m = 1, 2, \cdots, M\}$. Let m_0, $1 \leq m_0 < M$ be the approximate dimensionality of the distributed signal subspace, then the signal eigenvectors associated with $\{\lambda_1, \cdots, \lambda_{M_0}\}$ forms the signal eigenvector matrix given by

$$\boldsymbol{V}_s = [v_1, v_2, \cdots, v_{m_0}] \tag{IV.1}$$

The eigenvectors associated with $\{\lambda_{m_0+1}, \lambda_{m_0+2}, \cdots, \lambda_M\}$ forms the noise eigenvector matrix given by

$$\boldsymbol{V}_\nu = [\boldsymbol{v}_{m_0+1}, \boldsymbol{v}_{m_0+2}, \cdots, \boldsymbol{v}_M] \tag{IV.2}$$

Definition IV.1 *For matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of compatible dimensions, the orthogonality of $\boldsymbol{A}$ and $\boldsymbol{B}$ is defined by*

$$\boldsymbol{A}^\dagger \boldsymbol{B} = \boldsymbol{O} \tag{IV.3}$$

Since $\boldsymbol{R}_x$ is a Hermitian matrix, its eigenvectors $\{\boldsymbol{v}_m; m = 1, 2, \cdots, M\}$ are mutually orthogonal, i.e.,

$$\boldsymbol{V}_s^\dagger \boldsymbol{V}_\nu = \boldsymbol{O} \tag{IV.4}$$

In practice, we do not know $\boldsymbol{R}_x$. We only can estimate $\boldsymbol{R}_x$ by averaging over N samples so that

$$\hat{\boldsymbol{R}}_x = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{x}(n) \boldsymbol{x}^\dagger(n) \tag{IV.5}$$

which is asymptotically equal to $\boldsymbol{R}_x$. The eigenvalues and eigenvectors of $\hat{\boldsymbol{R}}_x$ are denoted respectively by $\{\hat{\mu}_1, \hat{\mu}_2, \cdots, \hat{\mu}_M\}$ and $\{\hat{\boldsymbol{v}}_1, \hat{\boldsymbol{v}}_2, \cdots, \hat{\boldsymbol{v}}_M\}$ where $\{\hat{\boldsymbol{v}}_m; m = 1, 2, \cdots, M\}$ are mutually orthogonal. We select $\{\hat{\boldsymbol{v}}_{m_0+1}, \hat{\boldsymbol{v}}_{m_0+2}, \cdots, \hat{\boldsymbol{v}}_M\}$ to be the estimated noise eigenvectors spanning the estimated noise subspace. We denote the estimated noise eigenvector matrix as

$$\hat{\boldsymbol{V}}_\nu = [\hat{\boldsymbol{v}}_{m_0+1}, \hat{\boldsymbol{v}}_{m_0+2}, \cdots, \hat{\boldsymbol{v}}_M] \tag{IV.6}$$

We now assume a particular distribution for the signal and hence obtain the corresponding signal correlation matrix $\boldsymbol{R}_{ss}$. We can rewrite the matrix $\boldsymbol{R}_{ss}$ in terms of its eigenvalues and eigenvectors, and approximate it in terms of its significant eigenvalues and associate eigenvectors,

$$\boldsymbol{R}_{ss} = \sum_{m=1}^{M} \lambda_m \boldsymbol{v}_m \boldsymbol{v}_m^\dagger \simeq \sum_{m=1}^{m_0} \lambda_m \boldsymbol{v}_m \boldsymbol{v}_m^\dagger = \boldsymbol{V}_s \boldsymbol{\Lambda}_s \boldsymbol{V}_s^\dagger \tag{IV.7}$$

where

$$\boldsymbol{\Lambda}_s = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{m_0}) \tag{IV.8}$$

Since the estimated data correlation matrix $\hat{R}_x$ asymptotically converges to R_x, therefore, the eigenvectors of $\hat{R}_x$ are asymptotically equal to the eigenvectors of R_x, i.e.,

$$\hat{v}_m \rightarrow v_m, \quad m = 1, 2, \cdots, M \tag{IV.9}$$

where "$\rightarrow$" denotes asymptotic convergence. Hence, the estimated noise eigenvector matrix $\hat{V}_\nu$ is orthogonal to signal eigenvector matrix V_s asymptotically.

There are various ways of utilizing the orthogonality of the signal eigenvector matrix and the noise eigenvector matrix, which result in different DOA estimation algorithms.

The DISPARE 1 Algorithm

Let us examine the weighted inner products of estimated noise eigenvectors and the eigenvectors of R_{ss},

$$\begin{aligned} \gamma_{mn} = \lambda_m^{1/2} \hat{v}_n^\dagger v_m, \qquad & n = m_0 + 1, m_0 + 2, \cdots, M \\ & m = 1, 2, \cdots, M \end{aligned} \tag{IV.10}$$

We note that γ_{mn} should be very small quantities since $\hat{v}_n$ and v_m are nearly orthogonal for $m \neq n$ and for $m = n$, i.e., $m, n \in \{m_0 + 1, \cdots, M\}$, we know λ_m are negligibly small, so that the weighted inner products are negligible quantities. Therefore, we can minimize the sum of the weighted inner products to locate the mean DOA $\bar{\phi}$ and the spread σ_ϕ. The DISPARE 1 algorithm can be written as

$$\begin{aligned} \{\hat{\bar{\phi}}, \hat{\sigma}_\phi\} &= \arg\min_{\bar{\phi}, \sigma_\phi} \sum_{m,n} |\, \gamma_{mn} \,|^2 \\ &= \arg\min_{\bar{\phi}, \sigma_\phi} \| \hat{V}_\nu^\dagger \left(\sum_{m=1}^{M} \lambda_m^{1/2} v_m \right) \|^2 \\ &= \arg\min_{\bar{\phi}, \sigma_\phi} \mathrm{tr}\{\hat{V}_\nu^\dagger R_{ss} \hat{V}_\nu\} \end{aligned} \tag{IV.11}$$

Eq.(IV.11) expresses the criterion of DISPARE 1. When the function $\mathrm{tr}\{\hat{V}_\nu^\dagger R_{ss} \hat{V}_\nu\}$ is minimized, the corresponding values of $\bar{\phi}$ and σ_ϕ will be the DISPARE 1 estimates of the mean DOA and signal spread.

The DISPARE 2 Algorithm

We now show that if the signal eigenvector matrix $\boldsymbol{V}_s$ and estimated noise eigenvector matrix $\hat{\boldsymbol{V}}_\nu$ are orthogonal, then the matrix product $\boldsymbol{V}_s^\dagger \boldsymbol{\Lambda}_s \boldsymbol{V}_s$ is also asymptotically orthogonal to $\hat{\boldsymbol{V}}_\nu$, i.e.,

$$\lim_{N\to\infty} \boldsymbol{V}_s \boldsymbol{\Lambda}_s \boldsymbol{V}_s^\dagger \hat{\boldsymbol{V}}_\nu \underset{a.s.}{=} \boldsymbol{O} \tag{IV.12}$$

where N denotes number of snapshots and $\boldsymbol{\Lambda}_s$ is given by Eq.(IV.8). Substituting Eq.(IV.7) into Eq.(IV.12), we have

$$\lim_{N\to\infty} \boldsymbol{R}_{ss} \hat{\boldsymbol{V}}_\nu \simeq \boldsymbol{O} \tag{IV.13}$$

Eq.(IV.13) is a close approximate to Eq.(IV.12) since the eigenvalues λ_m, $m = m_0+1, \cdots, M$, are negligible. Thus, we can conceive an algorithm in which we adjust the parameters $\bar{\phi}$ and σ_ϕ in $\boldsymbol{R}_{ss}$ until Eq.(IV.13) is satisfied. Since the Frobenius norm of a matrix is the measure of the sum of the square of each entry of the matrix, therefore, the DISPARE 2 algorithm can be written as

$$\{\hat{\bar{\phi}}, \hat{\sigma}_\phi\} = \arg\min_{\bar{\phi},\sigma_\phi} \| \boldsymbol{R}_{ss} \hat{\boldsymbol{V}}_\nu \|_F^2 \tag{IV.14}$$

where $\| \cdot \|_F^2$ denotes the Frobenius norm and $\boldsymbol{R}_{ss}$ is the correlation matrix of the distributed signal.

IV.2 The VEC-MUSIC Estimation

The intention of developing a method which may jointly estimate the DOA of a point source and a distributed source is to distinguish the point source from the distributed source even when the point source is merged into the distributed one. First, we go back to our discrete signal model and examine the case when one distributed and one point source are received without background noise. The received data by an array are given by

$$\begin{aligned} \boldsymbol{r}(n) &= \sum_{l=1}^{L} \boldsymbol{d}(\phi_l) p_l a_l(n) + \boldsymbol{d}(\phi_0) s_0(n) \\ &= \boldsymbol{D}\boldsymbol{P} \begin{bmatrix} a_1(n) \\ \vdots \\ a_L(n) \end{bmatrix} + \boldsymbol{d}(\phi_0) s_0(n) \end{aligned} \tag{IV.15}$$

The first term as has been observed in Eq.(II.12), is the received signal component corresponding to the distributed source. The second term is the point signal component in which $\boldsymbol{d}(\phi_0)$ is the array steering vector at the incident angle ϕ_0 of point source and $s_0(n)$ is the point signal waveform.

We see that the outer product of the received data $\boldsymbol{r}(n)$ is given by

$$\begin{aligned}
\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n) &= \boldsymbol{D}\boldsymbol{P}\begin{bmatrix} a_1(n) \\ \vdots \\ a_L(n) \end{bmatrix}[a_1^*(n)\cdots a_L^*(n)]\boldsymbol{P}\boldsymbol{D}^{\dagger} \\
&+ \boldsymbol{D}\boldsymbol{P}\begin{bmatrix} a_1(n) \\ \vdots \\ a_L(n) \end{bmatrix}s_0^*(n)\boldsymbol{d}^{\dagger}(\phi_0) \\
&+ \boldsymbol{d}(\phi_0)s_0(n)[a_1^*(n)\cdots a_L^*(n)]\boldsymbol{P}\boldsymbol{D}^{\dagger} \\
&+ \boldsymbol{d}(\phi_0)s_0(n)s_0^*(n)\boldsymbol{d}^{\dagger}(\phi_0) \qquad \text{(IV.16)}
\end{aligned}$$

We take the vec operation (see Section I) on $\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n)$ and evaluate the covariance matrix of $\text{vec}[\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n)]$, we have

$$\begin{aligned}
&\text{cov}\{\text{vec}(\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n))\} \\
&= E[\text{vec}(\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n))\text{vec}^{\dagger}(\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n))] \\
&- E[\text{vec}(\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n))]E[\text{vec}^{\dagger}(\boldsymbol{r}(n)\boldsymbol{r}^{\dagger}(n))] \\
&= (\boldsymbol{D}^*\boldsymbol{P}^2\boldsymbol{D}^T)\otimes(\boldsymbol{D}\boldsymbol{P}^2\boldsymbol{D}^{\dagger}) + (\boldsymbol{d}^*\boldsymbol{d}^T)\otimes(\boldsymbol{D}\boldsymbol{P}^2\boldsymbol{D}^{\dagger}) \\
&+ (\boldsymbol{D}^*\boldsymbol{P}^2\boldsymbol{D}^T)\otimes(\boldsymbol{d}\boldsymbol{d}^{\dagger}) + (\boldsymbol{d}^*\boldsymbol{d}^T)\otimes(\boldsymbol{d}\boldsymbol{d}^{\dagger}) \qquad \text{(IV.17)}
\end{aligned}$$

Henceforth, we omit the explicit dependence of $\boldsymbol{d}$ on the parameter ϕ.

From Eq.(II.13), we know

$$(\boldsymbol{D}\boldsymbol{P}^2\boldsymbol{D}^{\dagger}) = \sum_{l=1}^{L}\boldsymbol{d}(\phi_l)\boldsymbol{d}^{\dagger}(\phi_l)p_l^2 = \boldsymbol{R}_{ss} \qquad \text{(IV.18a)}$$

As we have mentioned in Section III.4, the approximate dimensionality of signal subspace of

the distributed signal is m_0, then the rank of matrix $\boldsymbol{R}_{ss}$ is approximately equal to m_0. Therefore, we have

$$\left.\begin{aligned} &\text{rank}\{(\boldsymbol{D}^*\boldsymbol{P}^2\boldsymbol{D}^T)\otimes(\boldsymbol{D}\boldsymbol{P}^2\boldsymbol{D}^\dagger)\} \simeq m_0^2 \\ &\text{rank}\{(\boldsymbol{D}^*\boldsymbol{P}^2\boldsymbol{D}^T)\otimes(\boldsymbol{d}\boldsymbol{d}^\dagger)\} \simeq m_0 \\ &\text{rank}\{(\boldsymbol{d}^*\boldsymbol{d}^T)\otimes(\boldsymbol{D}\boldsymbol{P}^2\boldsymbol{D}^\dagger)\} \simeq m_0 \\ &\text{rank}\{(\boldsymbol{d}^*\boldsymbol{d}^T)\otimes\boldsymbol{d}\boldsymbol{d}^\dagger)\} = 1 \end{aligned}\right\} \qquad \text{(IV.18b)}$$

If there are K_1 and K_2 distributed and point sources respectively, then the total dimensionality of the signal subspace before the Kronecker product is taken is $(K_1 m_0 + K_2)$. The dimensionality of the noise subspace is therefore $(M - K_1 m_0 - K_2)$. After the Kronecker product has been taken, the total dimensionality of the extended data space is M^2. The total dimensionality of the extended signal subspace is $(K_1 m_0 + K_2)^2$, whereas the total dimensionality of the extended pure noise subspace is $(M - K_1 m_0 - K_2)^2$. The remaining dimensionality is for a subspace which contains a mixture of noise and signals. This subspace is refered to as the *extended quasi noise subspace.*

For example, in the case when $M = 8$, $K_1 = K_2 = 1$, and $m_0 = 2$, then the dimensionality of the extended data space is $M^2 = 64$, the dimensionality of the extended signal subspace is $(m_0 + 1) = 9$, the dimensionality of the extended pure noise subspace is $(M - m_0 - 1)^2 = 25$, and the dimensionality of the extended quasi noise subspace is $64 - 9 - 25 = 30$.

Thus, the total rank of $\text{cov}\{\text{vec}(\boldsymbol{r}(n)\boldsymbol{r}^\dagger(n))\}$ in Eq.(IV.17) is approximately $m_0^2 + 2m_0 + 1 = (m_0+1)^2$. Here, $1 \le m_0 < M-1$, because we always require the number of sensors to be greater than the total signal dimension. However, for M sensors, the dimension of matrix $\text{cov}\{\text{vec}(\boldsymbol{r}(n)\boldsymbol{r}^\dagger(n))\}$ is $M \times M$. Hence, we can use the last few eigenvectors of the matrix $\text{cov}\{\text{vec}(\boldsymbol{r}(n)\boldsymbol{r}^\dagger(n))\}$, which are $\{\hat{\boldsymbol{u}}_{(m_0+1)^2+1}, \cdots, \hat{\boldsymbol{u}}_{M^2}\}$, corresponding to the eigenvectors spanning both the extended pure and quasi noise subspaces. We use $\hat{\boldsymbol{U}}_\nu$ to denote the estimated noise eigenvector matrix

$$\hat{\boldsymbol{U}}_\nu = [\hat{\boldsymbol{u}}_{(m_0+1)^2+1}, \cdots, \hat{\boldsymbol{u}}_{M^2}] \qquad \text{(IV.19)}$$

We let

$$\bar{\boldsymbol{d}} = \boldsymbol{d}^* \otimes \boldsymbol{d} \qquad \text{(IV.20)}$$

and

$$\bar{\boldsymbol{R}}_{ss} = (\boldsymbol{D}^* \boldsymbol{P}^2 \boldsymbol{D}^T) \otimes (\boldsymbol{D} \boldsymbol{P}^2 \boldsymbol{D}^\dagger) \tag{IV.21}$$

We note that $\bar{\boldsymbol{d}}$ and $\bar{\boldsymbol{R}}_{ss}$ only involve the point signal and distributed signal, respectively. Employing an idea similar to that of DISPARE 2 to find the DOA of the distributed signal, such that the VEC-MUSIC algorithm for the DOA estimation of a distributed source is given by

$$\{\hat{\bar{\phi}}, \hat{\sigma}_\phi\} = \arg \min_{\bar{\phi}, \sigma_\phi} \| \bar{\boldsymbol{R}}_{ss} \hat{\boldsymbol{U}}_\nu \|_F^2 \tag{IV.22}$$

where $\| \cdot \|_F^2$ is the Frobenius norm and $\bar{\boldsymbol{R}}_{ss}$ is an explicit function of the parameters to be estimated. Eq.(IV.22) describes a criterion for the estimation of the mean DOA and the spread of the distributed signal.

On the other hand, to estimate a point source, we can utilize the component $\bar{\boldsymbol{d}}$ which as we have noted, involves only the point source. Thus, we can employ the orthogonality between $\bar{\boldsymbol{d}}$ and $\boldsymbol{U}_\nu$, and formulate a projection onto the estimated noise subspace such that

$$\hat{\boldsymbol{P}}_\nu = \hat{\boldsymbol{U}}_\nu \hat{\boldsymbol{U}}_\nu^\dagger \tag{IV.23}$$

Applying this projector on $\boldsymbol{d}$, and changing the parameter ϕ in the steering vector, we can locate the DOA of the point source by

$$\hat{\bar{\phi}} = \arg \min_{\bar{\phi}} \bar{\boldsymbol{d}}^\dagger \hat{\boldsymbol{U}}_\nu \hat{\boldsymbol{U}}_\nu^\dagger \bar{\boldsymbol{d}} \tag{IV.24}$$

Eq.(IV.24) describes the criterion for estimating the DOA of a point source signal. The pair of equations (IV.22) and (IV.24) are called the VEC-MUSIC algorithm. We can choose various combinations of the pure and quasi noise eigenvectors in Eq.(IV.19) to form the projector $\hat{\boldsymbol{P}}_\nu$ resulting in a more flexible VEC-MUSIC algorithm.

V Computer Simulation Results

In this section, computer simulations are conducted to examine the effectiveness of the proposed algorithms in Section IV when they are applied to the different cases involving distributed sources and point sources. Two forms of distribution for the distributed source spectrums are tested – the uniform distribution and the Gaussian distribution.

For the algorithms of DISPARE 1 and DISPARE 2, a two-dimensional search is conducted with the initial estimates chosen around the true DOA's and the true spreads. For the VEC-MUSIC algorithm, by assuming the width of the spread is known, a one-dimensional search is conducted for the mean DOA estimation of the distributed signal; for estimating the DOA of the point signal, only one-dimensional search is involved.

The main difference between VEC-MUSIC and the DISPARE algorithms is the ability of VEC-MUSIC to distinguish point sources from distributed sources. This is because the VEC-MUSIC algorithm projects the quantity $\bar{d}$ onto the estimated noise subspace when searching for a point source, and a different quantity $\bar{R}_{ss}$ onto the estimated noise subspace when searching a distributed source. Furthermore, the VEC-MUSIC has the additional flexibility of the choice of noise subspaces onto which these quantities can be projected. Fig.V.1 and Fig.V.2 shows the spatial spectra of the VEC-MUSIC for the distributed signal and the point signal searches respectively. A 4-sensors uniform linear array with half signal source wavelength spacing is used, and all sensors are assumed to be omnidirectional with unity gain. The Gaussian distributed and the point signals are arriving at $-10°$ and $30°$ respectively, with the power of point signal being $2dB$ higher than the power of distributed one and without additive noise. The noise subspace chosen here is spanned by the extended noise eigenvectors. As can be observed, when the distributed signal is being located in Fig.V.1, the point source is relatively small compared to the peak caused by the distributed source. On the other hand, when the point source is being located in Fig.V.2, the peak caused by the distributed source is completely negligible.

We now examine the accuracy of the DOA estimation provided by the various methods. The following study cases examine the performance of the three methods DISPARE 1, DISPARE 2, and VEC-MUSIC under different environments. Since both DISPARE 1 and 2 are unable to distinguish a point source from a distributed one, we do not address the ability of signal distinction in these examples. Rather, we only compare the accuracy of the estimates obtained using these methods.

In each case, an 8-sensor uniform linear array with half signal source wavelength spacing is used, and again, all sensors are assumed to be omnidirectional with unity gain. Additive sensor noise is assumed to be a spatially white Gaussian process with zero mean.

Case I: Uniformly Distributed Signal Source

The first case we study is the situation in which only one distributed source arrives at $-10°$ with respect to the normal of the array. Two numerical experiments are carried out in which the total widths of the signal are 13.4° and 6.4° respectively. All angles here are physical angles. By using Eq.(III.9a), the dimensions of signal subspace are equal to 2 and 1 respectively for each of the spread signal source. Monte-Carlo simulations of 50 independent trials with 200 snapshots are performed for signal-noise ratio (SNR) from $0dB$ to $50dB$ with a $10dB$ step to evaluate the standard deviation (SD) of the mean DOA estimate. Fig.V.3 and Fig.V.4 show the SD of the mean DOA estimates in the two cases versus SNR for the three proposed methods for each spread. From Fig.V.3, we can see that DISPARE 1 and DISPARE 2 have similar performance, both being slightly better than VEC-MUSIC when SNR is over $20dB$. When the SNR is lower than $20dB$, DISPARE 1 shows more severe threshold effect than DISPARE 2 and VEC-MUSIC. Fig.V.4 shows that both DISPARE 1 and DISPARE 2 have same estimation errors and perform better than VEC-MUSIC for SNR from $0dB$ to $50dB$.

The second case is a situation where a point signal source arrives at $-30°$ and a distributed source at $-10°$ with respect to the normal of the array. Two numerical experiments are designed for the point source arriving together with a distibuted source the width of distribution of which are the same as those mentioned above. For this case, the dimension of the signal subspaces are

respectively 3 and 2. Again, 50 independent trials with 200 snapshots are carried out to evaluate the SD of the mean DOA of the distributed source and the DOA of the point source for SNR from $0dB$ to $50dB$ with a $10dB$ step. Fig.V.5 shows the SD of the estimate of the mean DOA of a distributed signal with the spread width being 13.4°. We can see that DISPARE 2 performs better than VEC-MUSIC and both performance are better than the performance of DISPARE 1. When the SNR is lower than $5dB$, VEC-MUSIC performs better than DISPARE 2. Fig.V.6 shows the SD of the estimate of the DOA of the point source. Both DISPARE 1 and DISPARE 2 have better performance than VEC-MUSIC, and when SNR increases, DISPARE 2 gradually outperforms DISPARE 1. Fig.V.7 and Fig.V.8 show the SD of the estimates of mean DOA of a distributed source with spread width being 6.4° and the DOA of a point source. DISPARE 1 and DISPARE 2 have similar performance and perform better than VEC-MUSIC.

Case II: Gaussian Distributed Signal Source

Parallel to the Case I, a set of numerical experiments are implemented under the same scenarios except the distributed signal source is of Gaussian distribution. For Gaussian distributed source, about 99% energy are within $[\bar{\phi} - 3\sigma_\phi, \bar{\phi} + 3\sigma_\phi]$, so σ_ϕ is chosen as one third of the width of the spread at each side of mean DOA. Fig.V.9 and Fig.V.10 are the SD of mean DOA estimates for $\sigma_\phi = 2.2°$ and $\sigma_\phi = 1.1°$, respectively. It is observed in Fig.V.9 that while both DISPARE 2 and VEC-MUSIC yield results worse than those in Fig.V.3 for a uniformly distributed source, DISPARE 1 yields totally unacceptable estimates under all SNR. On the other hand for a narrow signal spread, Fig.V.10 shows that all these methods yield good estimates, with VEC-MUSIC being slightly worse than either of the algorithms. Fig.V.11 to Fig.V.12 are the SD of the estimates of mean DOA of the distributed signal with $\sigma_\phi = 1.1°$, and the DOA of the point signal for all three proposed methods. Here in Fig.V.11, VEC-MUSIC has the best performance for high SNR while DISPARE 1 and DISPARE 2 have similar performance when locating the mean DOA of the distributed signal. On the other hand, for the location of the point signal, only DISPARE 2 and VEC-MUSIC provide acceptable estimates with DISPARE 2 marginally more accurate than VEC-MUSIC. The error in DISPARE 1 is too large to be acceptable.

Case III: Signal Distribution Model Sensitivity

We have seen that in order to apply our proposed methods to estimate the DOA of a distributed signal source, a certain type of distribution has to be first assumed. For practical situation, the actual distribution may not be known. A group of numerical experiments are set up to examine the performance deterioration of the three methods when the distribution is assumed wrongly. Fig.V.13 shows the SD of the mean DOA estimates of the three methods when a uniformly distributed signal source with spread $\sigma_\phi = 13.4°$ impinges upon the array from $-10°$, but a Gaussian distribution has been assumed in the estimation. It is observed from Fig.V.13 that all these methods yield very large errors. Fig.V.14 shows the performance of the three methods estimating the DOA of a Gaussian distributed signal with $\sigma_\phi = 2.2°$ impinging upon the array at $-10°$, but a uniformly distributed signal has been assumed in the estimation. In this case, both DISPARE 2 and VEC-MUSIC yield estimates with acceptable errors. However, the error in the estimate using DISPARE 1 is much larger for all SNR.

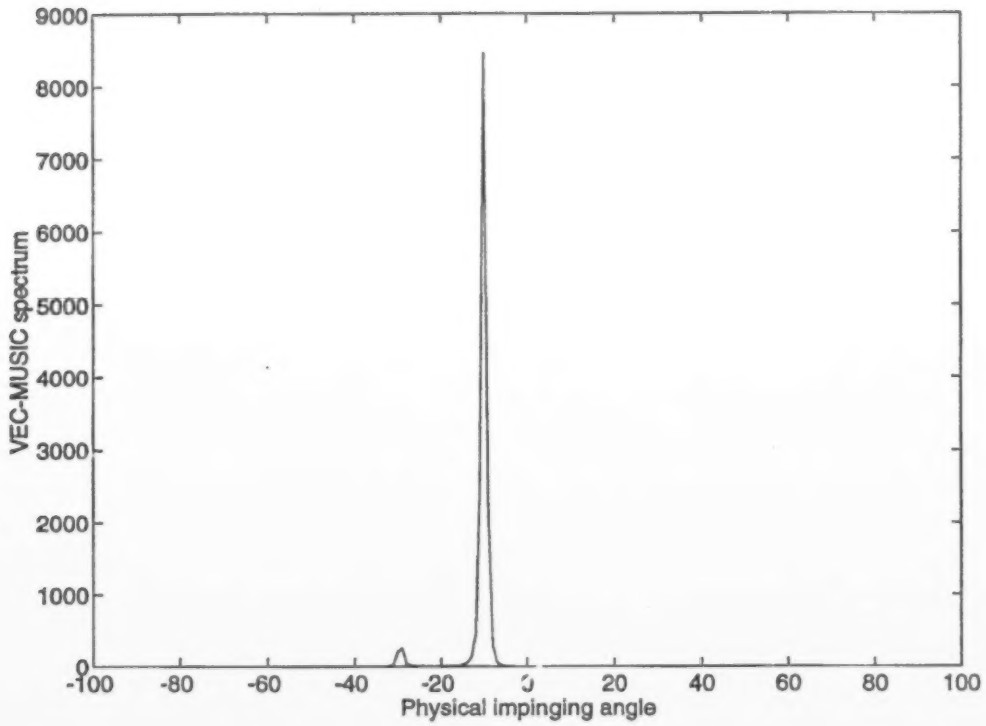


Figure V.1: VEC-MUSIC: search for a distributed source at $-10°$

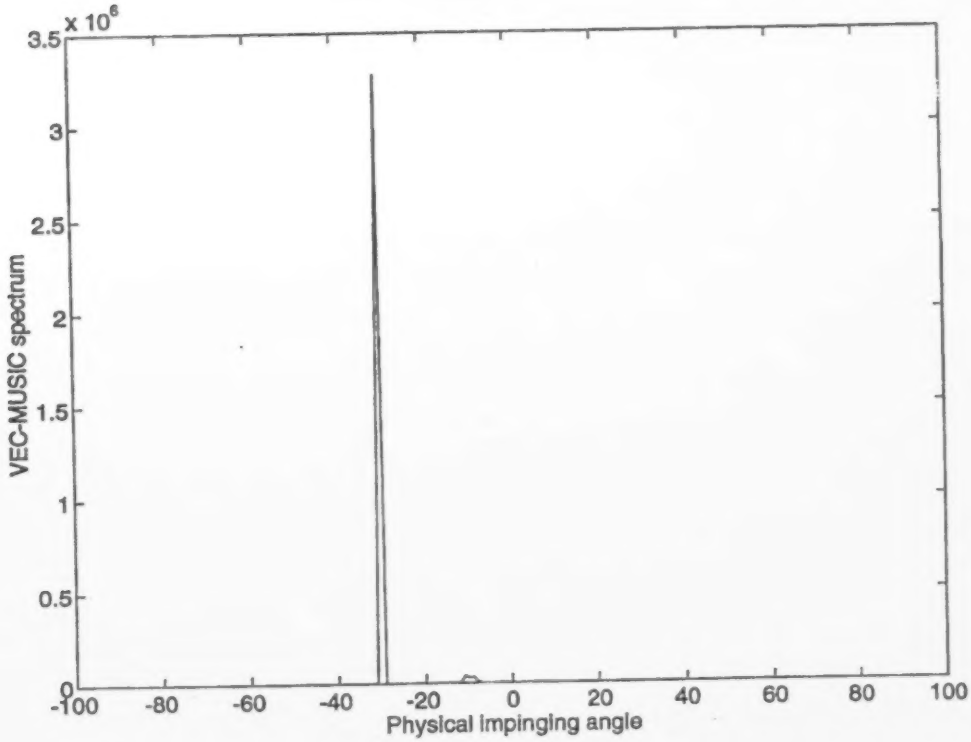


Figure V.2: VEC-MUSIC: search for a point source at $-30°$

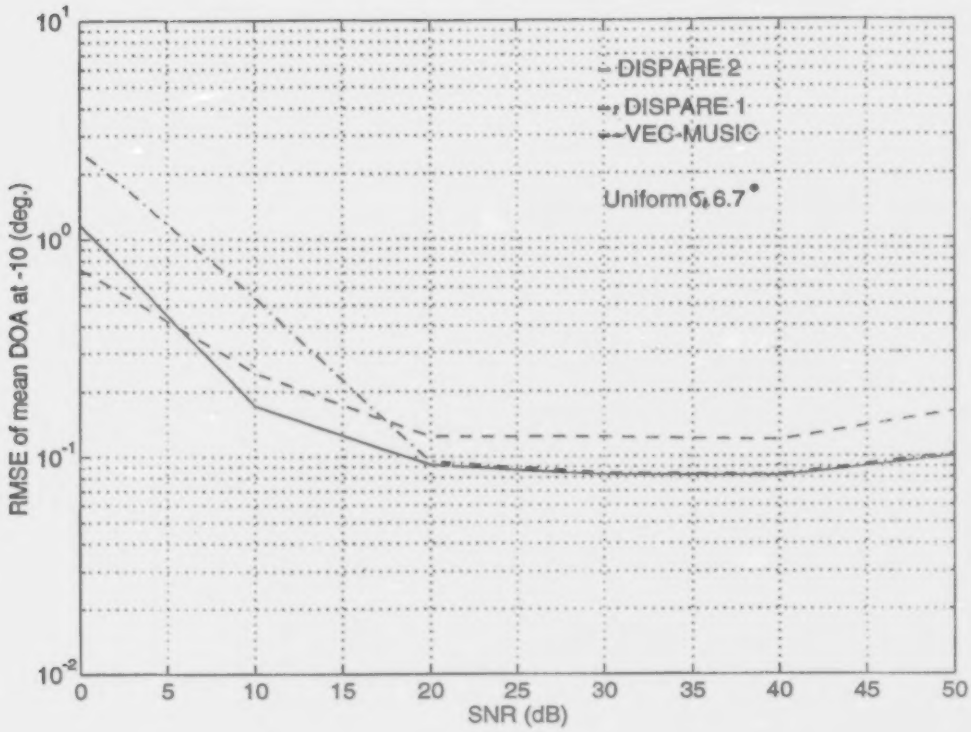


Figure V.3: The estimation errors of mean DOA of a uniformly distributed source with $\sigma_\phi = 6.7°$

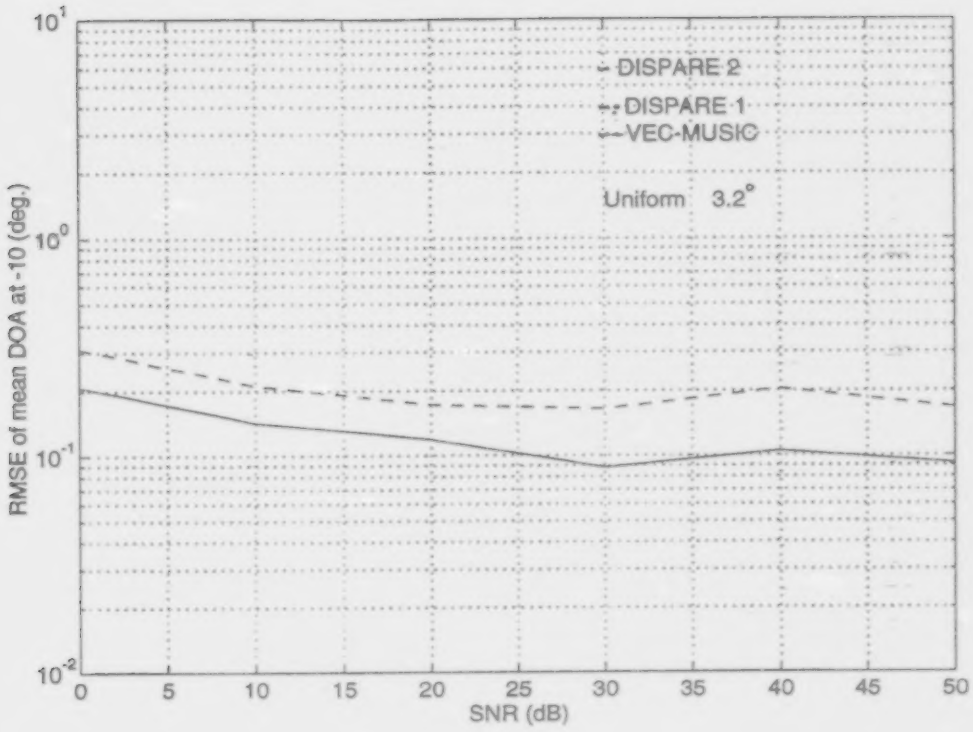


Figure V.4: The estimation errors of mean DOA of a uniformly distributed source with $\sigma_\phi = 3.2°$

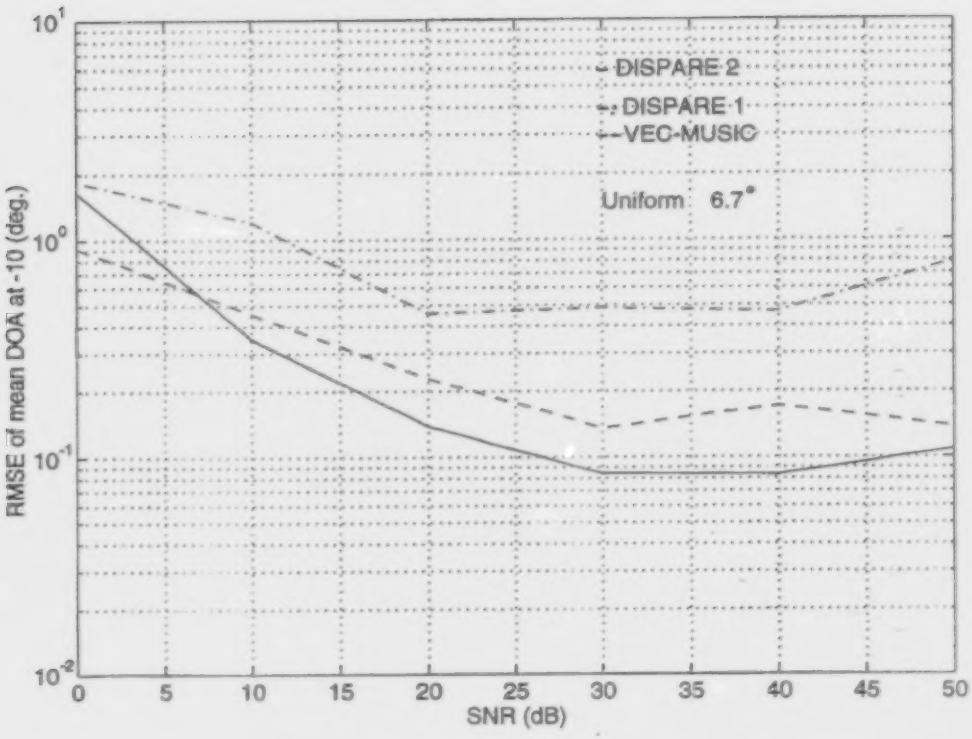


Figure V.5: The estimation errors of mean DOA of a uniformly distributed source with $\sigma_\phi = 6.7°$

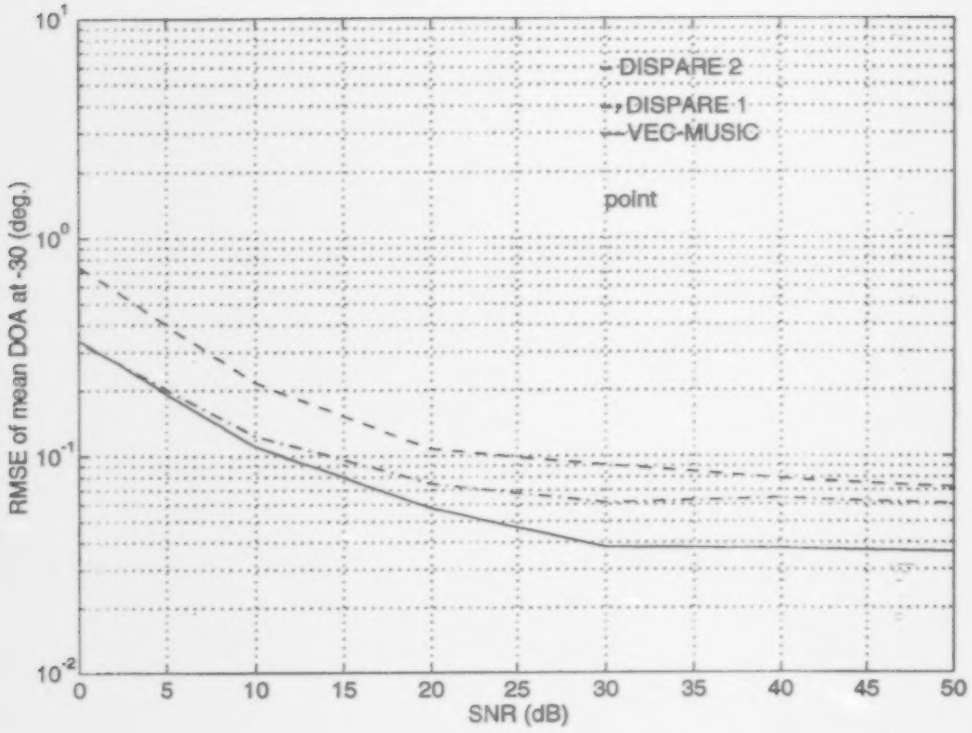


Figure V.6: The estimation errors of DOA of a point source

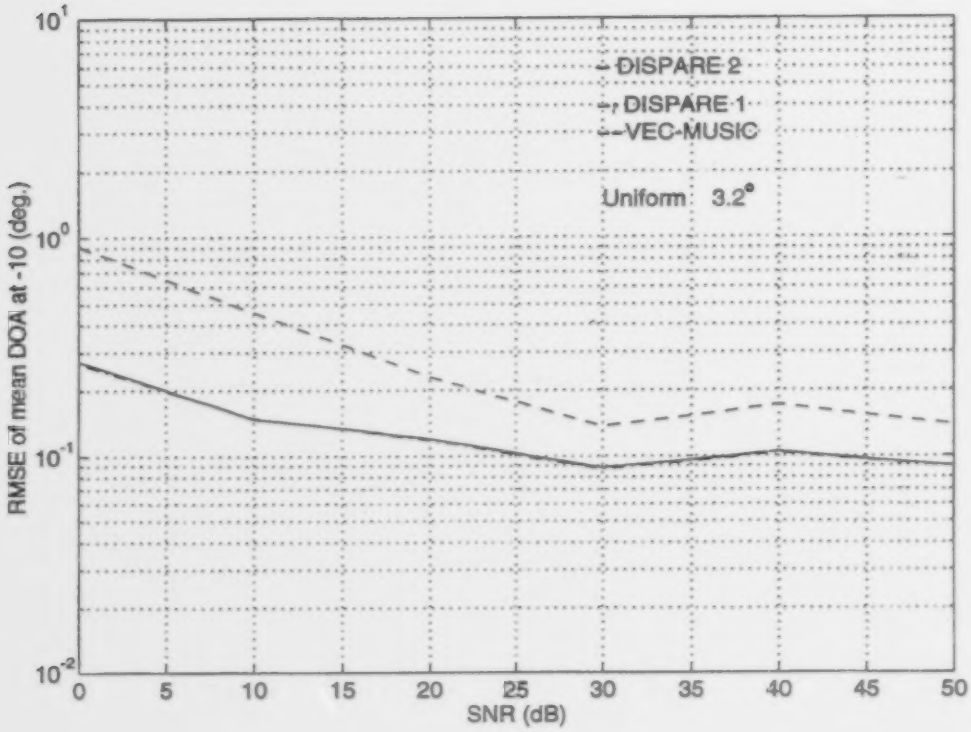


Figure V.7: The estimation errors of mean DOA of a uniformly distributed source with $\sigma_\phi = 3.2°$

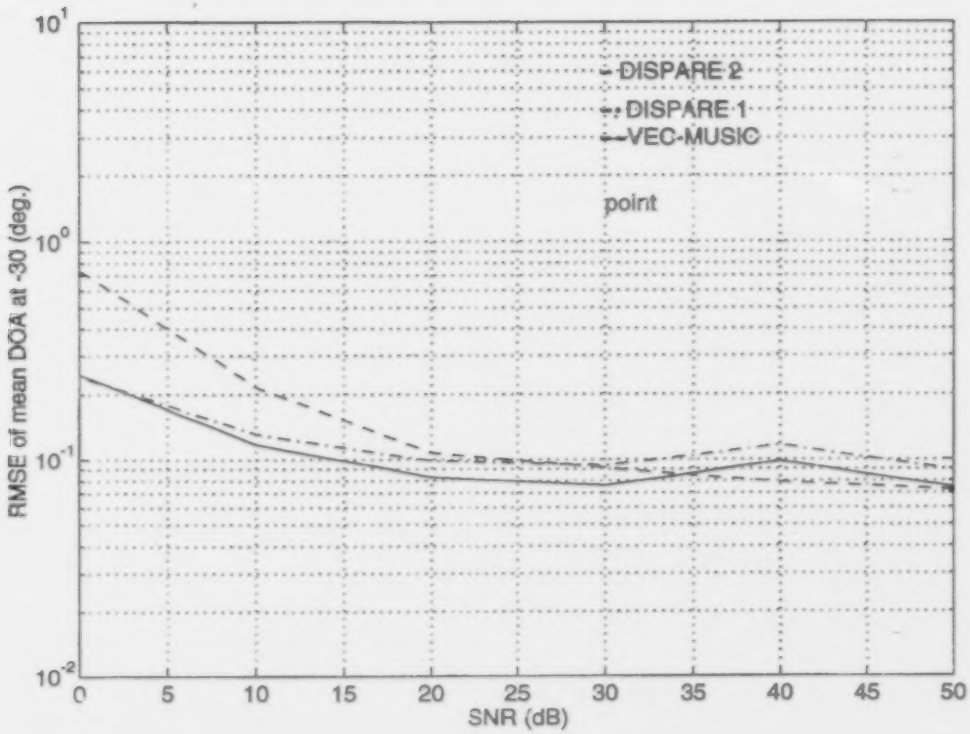


Figure V.8: The estimation errors of DOA of a point source

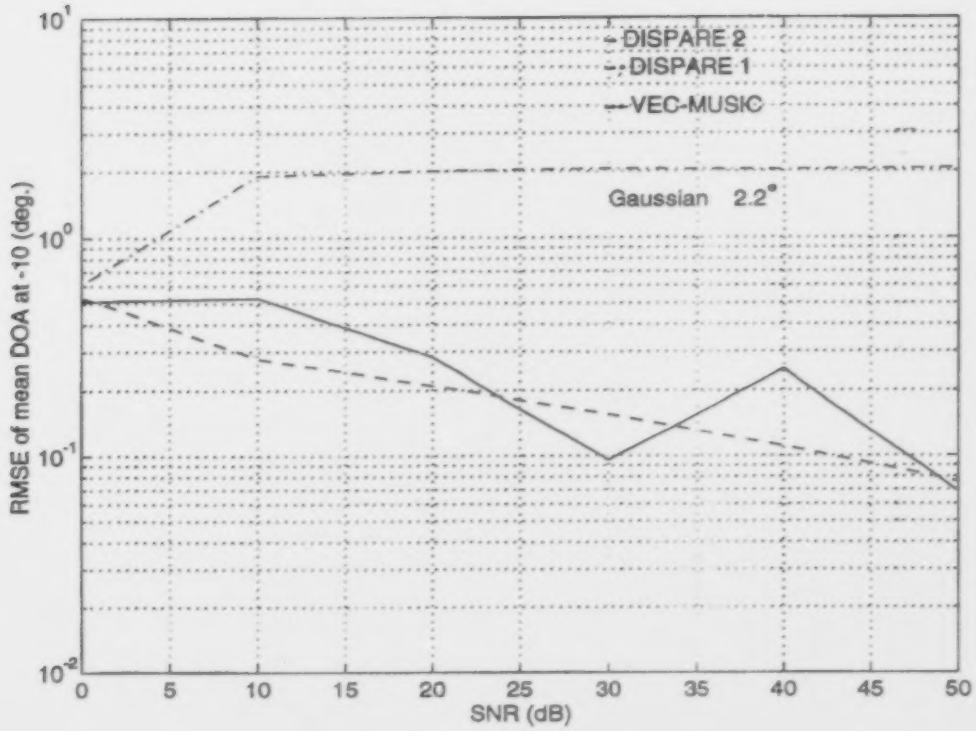


Figure V.9: The estimation errors of mean DOA of a Gaussian distributed source with $\sigma_\phi = 2.2°$

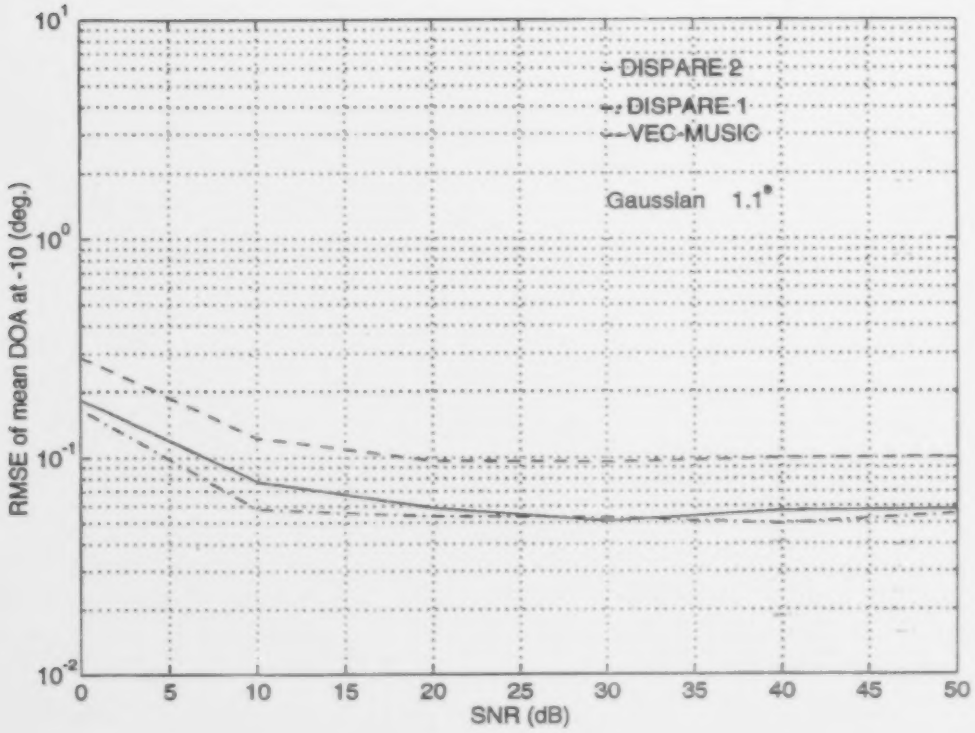


Figure V.10: The estimation errors of mean DOA of a Gaussian distributed source with $\sigma_\phi = 1.1°$

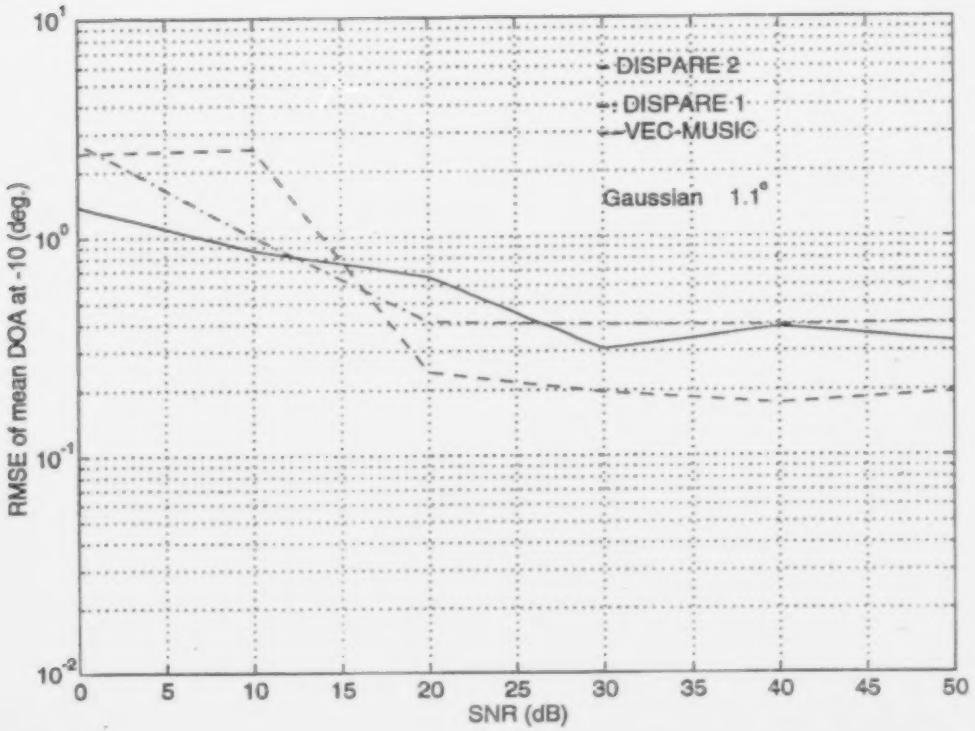


Figure V.11: The estimation errors of mean DOA of a Gaussian distributed source with $\sigma_\phi = 1.1°$

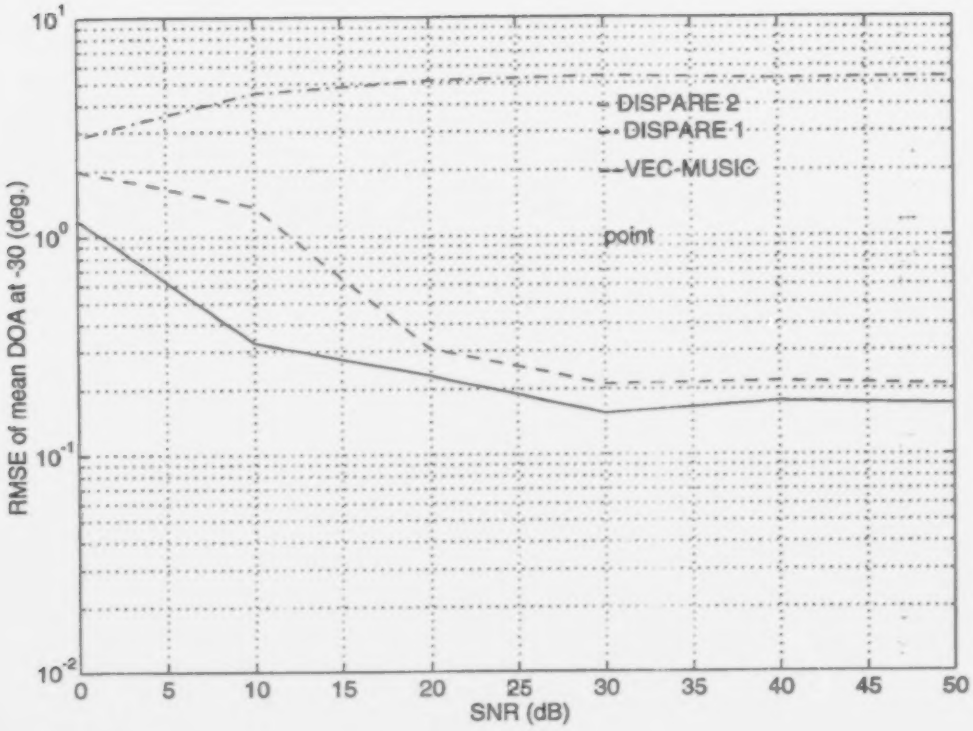


Figure V.12: The estimation errors of DOA of a point source

Case IV: Decrease in Number of Snapshots

Finally, the computer simulations in Case I and Case II are repeated for fewer number of snapshots. Fig.V.15 to Fig.V.18 demonstrate the results for 50 and 20 snapshots, respectively. It is observed, as expected, that all the methods deteriorate. The smaller is the number of snapshots the greater is the performance deterioration.

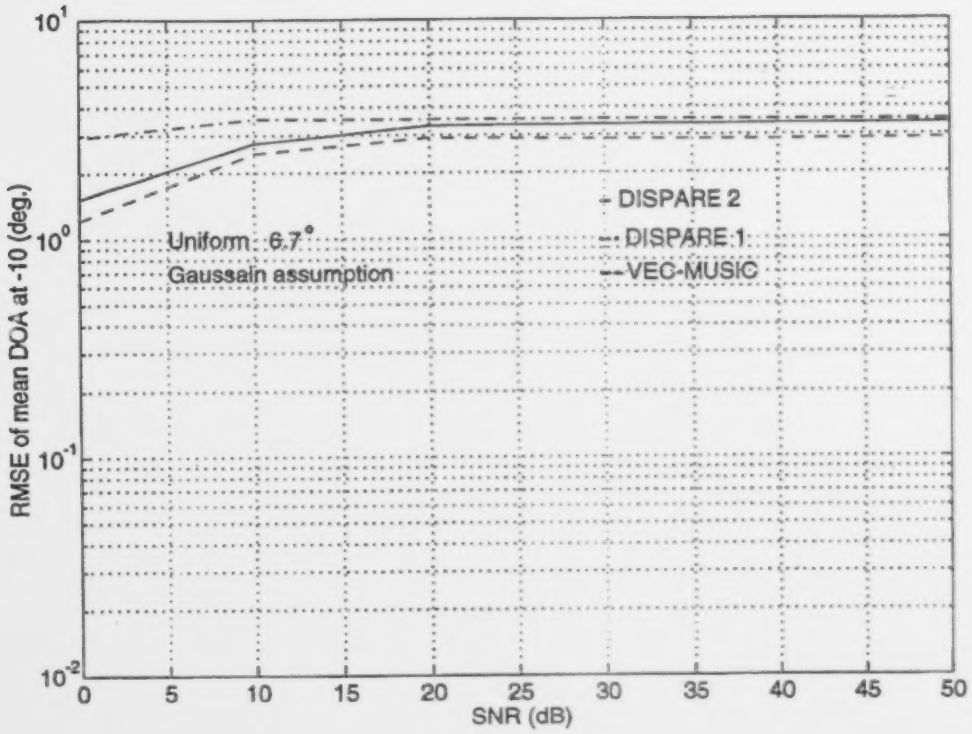


Figure V.13: The estimation errors of mean DOA of a uniformly distributed source under the assumption of a Gaussian distribution

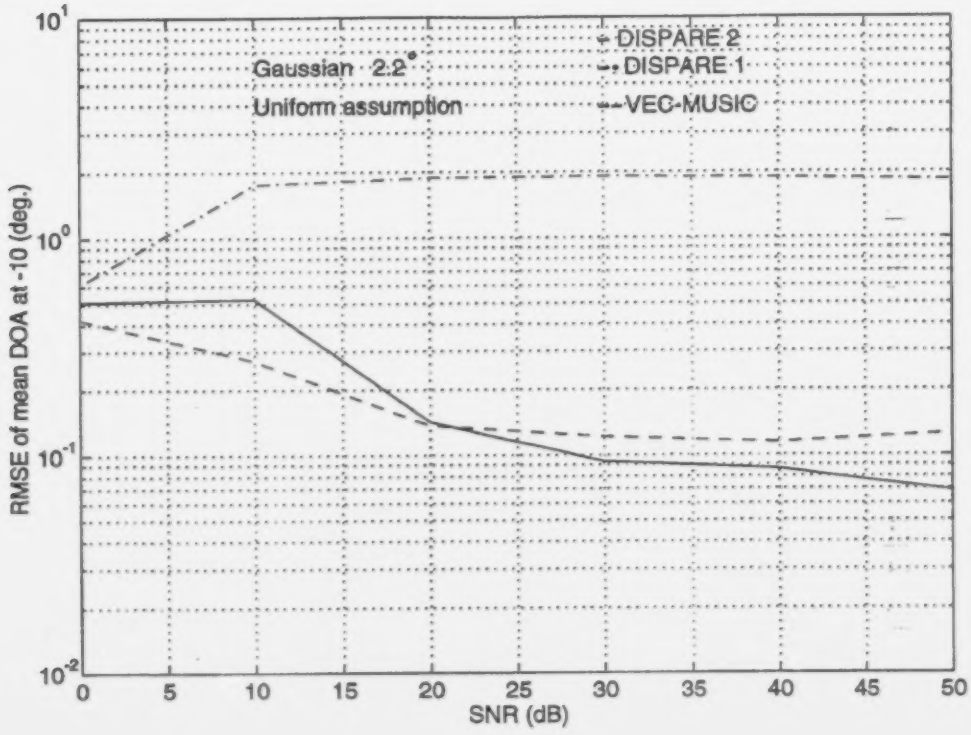


Figure V.14: The estimation errors of mean DOA of a Gaussian distributed source under the assumption of a uniform distribution

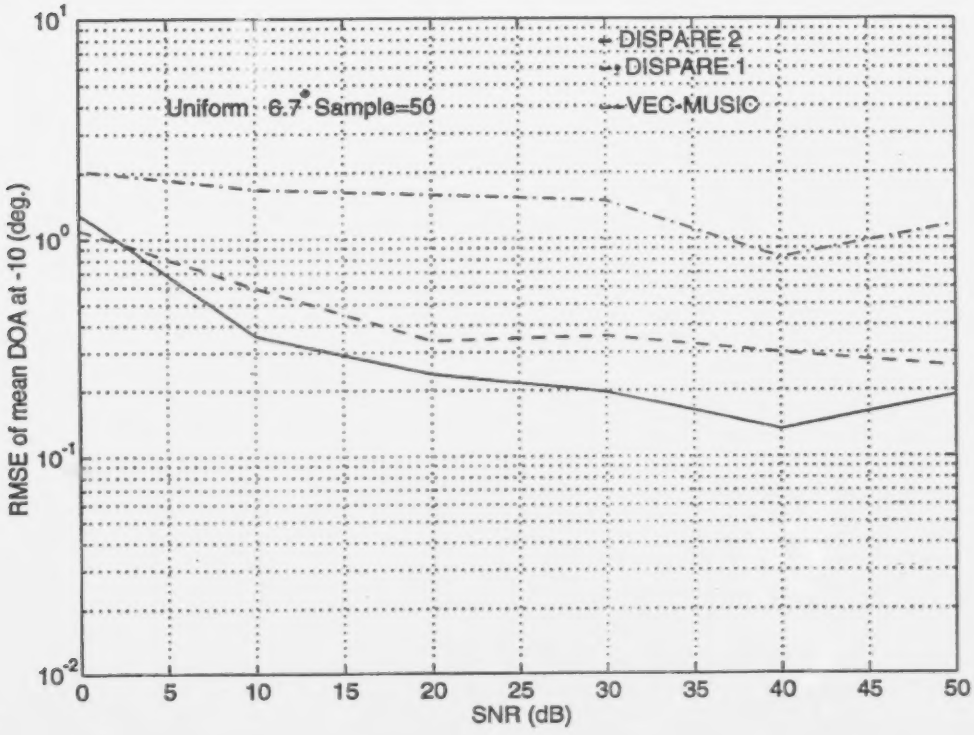


Figure V.15: The estimation errors of mean DOA of a uniformly distributed source ($\sigma_\phi = 6.7°$) with 50 snapshots

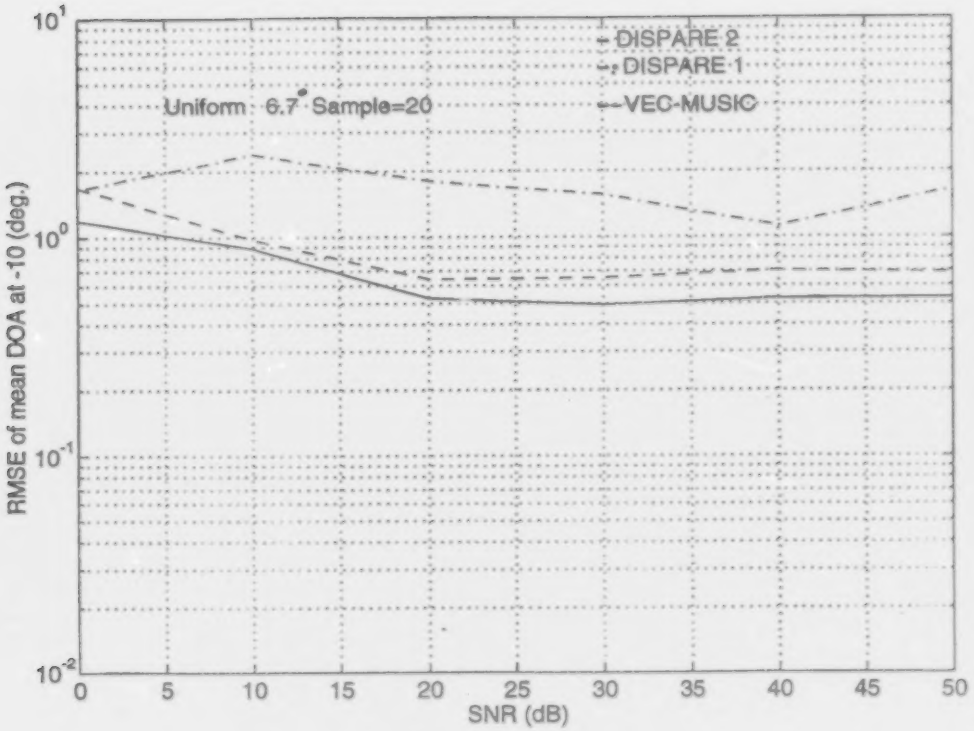


Figure V.16: The estimation errors of mean DOA of a uniformly distributed source ($\sigma_\phi = 6.7°$) with 20 snapshots

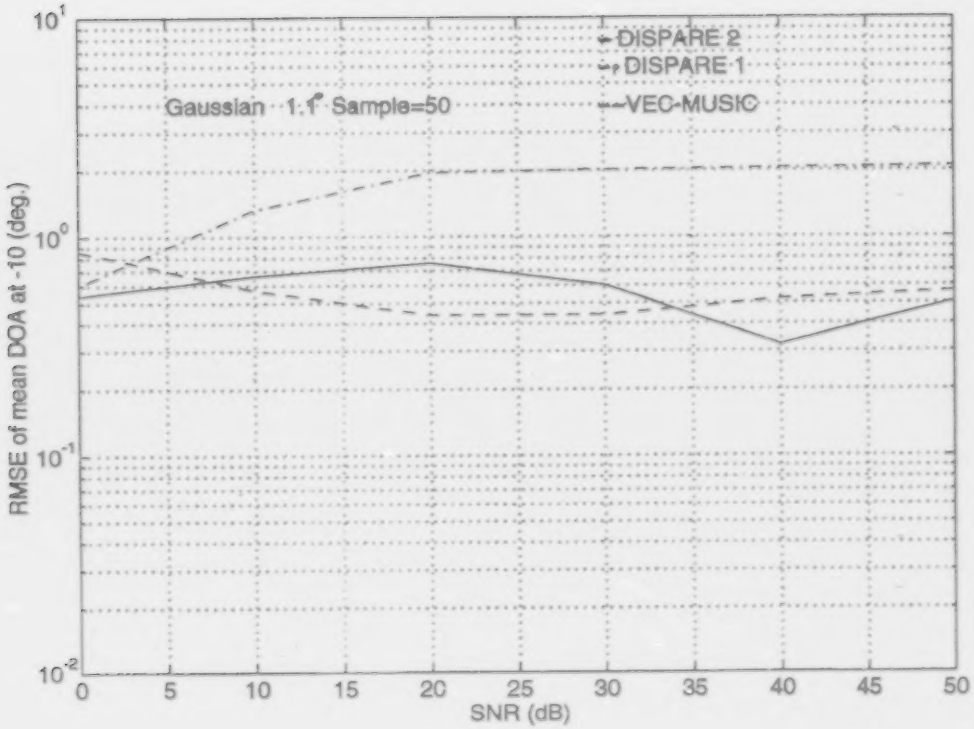


Figure V.17: The estimation errors of mean DOA of a Gaussian distributed source ($\sigma_\phi = 1.1°$) with 50 snapshots

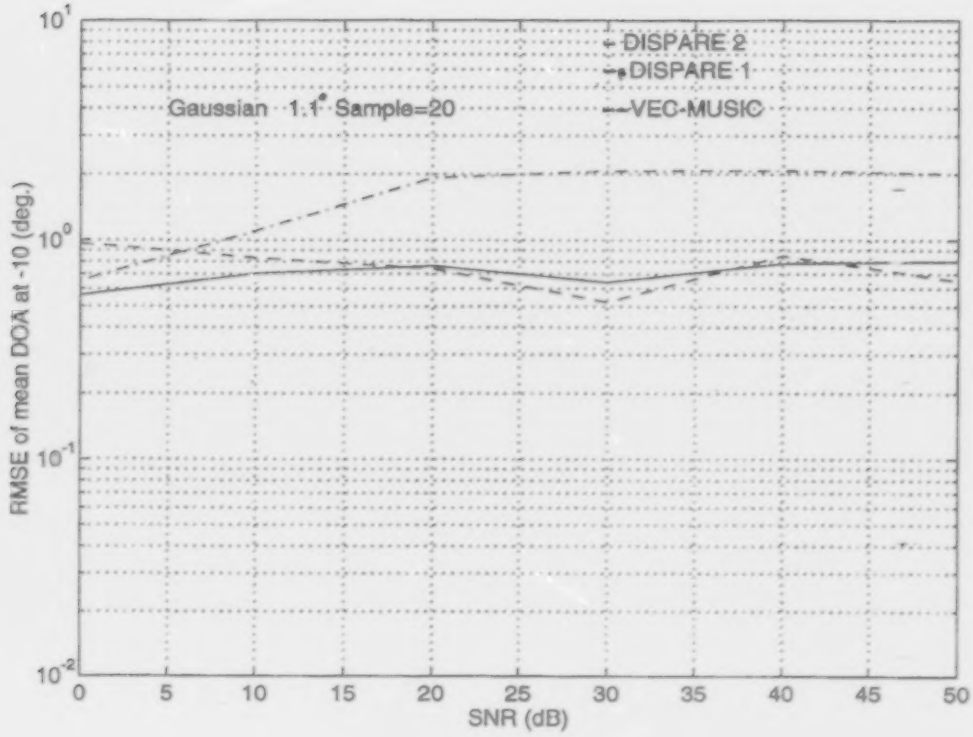


Figure V.18: The estimation errors of mean DOA of a Gaussian distributed source ($\sigma_\phi = 1.1°$) with 20 snapshots

VI Discussions and Conclusions

In this report, we have examined the problem of estimating the DOA of spatially distributed source in a white Gaussian noise environment. A parametric distributed signal correlation matrix for a distributed signal has been derived and the important properties of the distributed signal correlation matrix have been examined. Although the correlation matrix for a distributed signal is of full rank, it has been shown that the dimension of the signal subspace can be approximated to m_0, a number much lower than M, the number of sensors. Using this fact, it is possible to partition the signal and noise subspaces. Based on the signal model and the properties of the correlation matrix, we have developed the distributed signal parameter estimates DISPARE 1 and DISPARE 2 which utilize the orthogonality between the signal eigenvectors and the estimated noise eigenvectors. On the other hand, by using the orthogonality between the extended signal and noise eigenvectors, we have proposed the VEC-MUSIC algorithm to estimate the mean DOA of a distributed source and the DOA of a point source by distinct criteria. Clearly, the two kinds of sources can be distinguished. It is worth to mention here that by using the extended noise eigenvectors, VEC-MUSIC has the advantage of being flexible to choose various estimated noise eigenvector matrices from $\hat{\boldsymbol{U}}_{\nu} = [\hat{\boldsymbol{u}}_{(m_0+1)^2+1}, \cdots, \hat{\boldsymbol{u}}_{M^2}]$ to $\hat{\boldsymbol{U}}_{\nu p} = [\hat{\boldsymbol{u}}_{(M-m_0+1)^2}, \cdots, \hat{\boldsymbol{u}}_{M^2}]$, we name them estimated extended noise eigenvector matrix and estimated extended pure noise eigenvector matrix, respectively.

From the simulation results, we can see that DISPARE 2 and VEC-MUSIC have similar performance and out-perform DISPARE 1 in almost all scenarios. DISPARE 1 and DISPARE 2 have approximately the same computational complexity; while VEC-MUSIC is computationally more intensive. By using the estimated extended pure noise eigenvector matrix, we have found that we can reduce the computational load of VEC-MUSIC greatly because of the avoidance of the eigendecomposition of an extended data covariance matrix. We can evaluate the estimated extended pure noise eigenvector matrix by using the Kronecker product of the estimated noise eigenvectors of the original data correlation matrix. This is because the Kronecker product maintains the orthogonality between vectors.

From the numerical experiments, we also observe that the assumption of uniform distribution of signal source renders all methods more robust to model error. As well, if the number of snapshots decreases, the performance of all these methods deterioarate.

A Appendix A

A.1 The Discrete Prolate Spheroidal Sequences

The *discrete prolate spheroidal sequences* (DPSS) [75] $\{q_m^{(i)}(W)\}$ is defined as the real solution to the system of equations

$$\lambda_m(W)q_m^{(i)}(W) = \sum_{j=1}^{M} \frac{\sin 2\pi W(i-j)}{\pi(i-j)} q_m^{(i)}(W), \quad i = \pm 1, \pm 2, \cdots \tag{A.1}$$

for each $m = 1, 2, \cdots, M$, where $q_m^{(i)}$ denotes the ith element of sequence q_m. It is understood here, of course, that when $i = j$ the expression $\frac{\sin 2\pi W(i-j)}{\pi(i-j)}$ has the value $2W$.

We see here $\lambda_m(W)$, $m = 1, 2, \cdots, M$ are the eigenvalues of an $M \times M$ matrix $\boldsymbol{R}_{M,W}$ with the ijth entry

$$R_{M,W}(i,j) = \frac{\sin 2\pi W(i-j)}{\pi(i-j)}, \quad i, j = 1, 2, \cdots, M \tag{A.2}$$

and M vectors obtained by index-limiting the DSPP $\{q_m^i(W)\}$, $m = 1, 2, \cdots, M$, to the index set $(1, M)$ are the M corresponding eigenvectors of the matrix $\boldsymbol{R}_{M,W}$.

The DPSS's are doubly orthogonal, i.e.,

$$\sum_{i=1}^{M} q_m^{(i)}(W)q_n^{(i)}(W) = \lambda_m(W)\delta_{mn} \tag{A.3a}$$

$$\sum_{i=-\infty}^{\infty} q_m^{(i)}(W)q_n^{(i)}(W) = \delta_{mn} \tag{A.3b}$$

for $m, n = 1, 2, \cdots, M$. The DPSS $\{q_m^{(i)}(W)\}$, $i = \pm 1, \pm 2, \cdots$, are limited to $(-W, W)$ in frequency domain, the energy concentration of index-limited DPSS $\{q_m^{(i)}(W)\}$, $m = 1, 2, \cdots, M$, is given by

$$\lambda_m(W) = \frac{\sum_{i=1}^{M} \mid q_m(W)^{(i)} \mid^2}{\sum_{i=-\infty}^{\infty} \mid q_m(W)^{(i)} \mid^2} \quad m = 1, \cdots, M \tag{A.4}$$

which is the ratio of the energy of index-limited DPSS's and the energy of entire DPSS's.

Fig.A.1 to Fig.A.3 show the eigenvalues $\lambda_m(W)$ for a different W. We can see that for fixed W, $\lambda_1(W) \geq \lambda_2(W) \geq \cdots \geq \lambda_M(W)$; and when the length M of the index-limited DPSS's increases, more eigenvalues $\lambda_m(W)$ have significant value because more energy is involved.

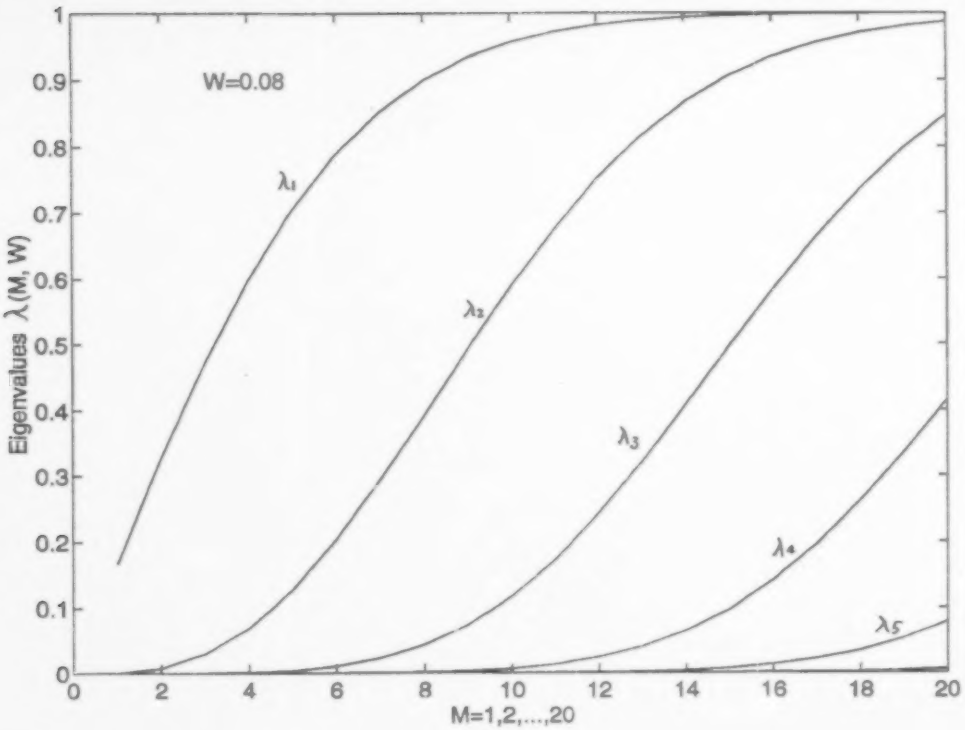


Figure A.1: The eigenvalues $\lambda_m(M)$ for $W = 0.08$

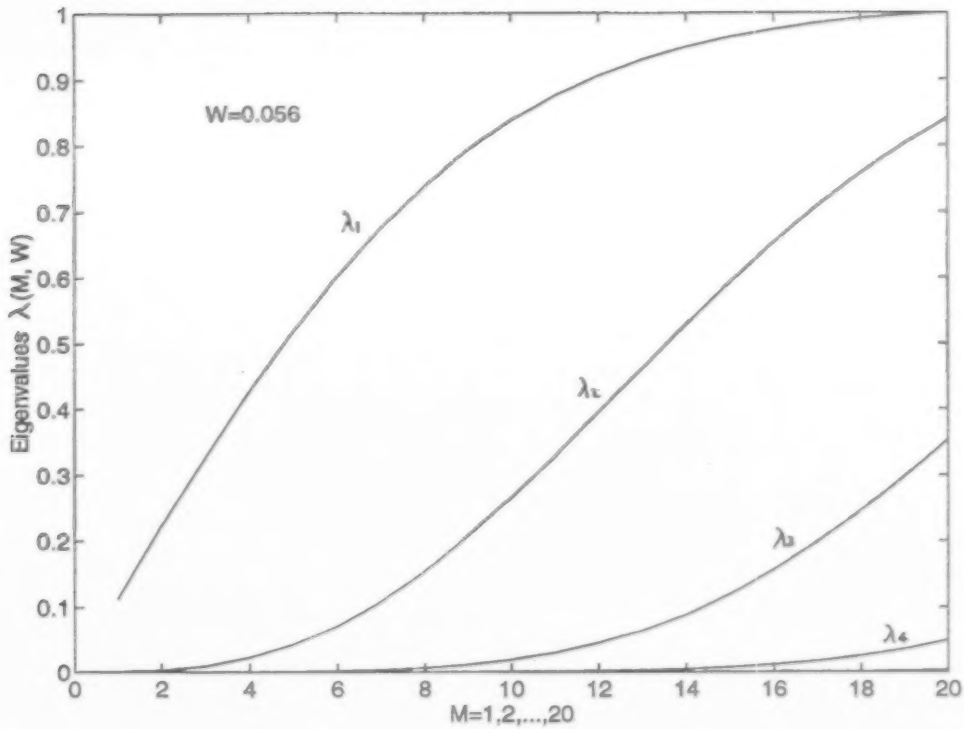


Figure A.2: The eigenvalues $\lambda_m(M)$ for $W = 0.056$

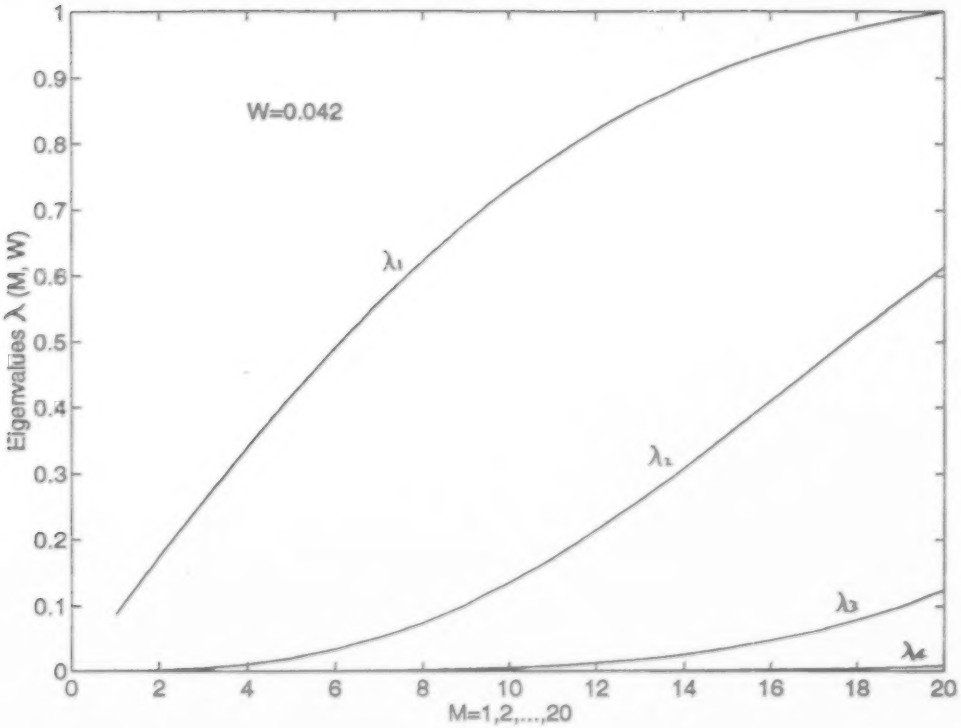


Figure A.3: The eigenvalues $\lambda_m(M)$ for $W = 0.042$

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(U) In this report, the problem of estimating the directions of arrival (DOA) of distributed signals is examined. A mathematical model is first established by making some reasonable assumptions. The correlation matrix of a distributed signal is then derived. Examples of signals with different distributions are presented. The important properties of the correlation matrix are studied revealing that even if the matrix is of full rank (being equal to the number of sensors) which renders the conventional high resolution array processing methods inapplicable in this case, the dimensionality of the signal subspace can be approximated to a number usually much smaller than the number of sensors. From this observation and utilizing the orthogonality of the signal and noise subspaces, two groups of algorithms (DISPARE and VEC-MUSIC) have been developed. Several computer simulation examples are carried out to examine the performance of the algorithms under various environments. It is observed that, in general, the algorithm DISPARE 1 may not be robust in the sense that it may yield unacceptable results under some particular conditions. On the other hand, DISPARE 2 generally yields the least estimation error, being slightly better than VEC-MUSIC. However, the VEC-MUSIC algorithms, in general, have the additional flexibility of choosing the noise subspace (called the extended pure and extended quasi noise subspaces) as well as the ability of distinguishing a distributed source from a point source.

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TITLE:(U) DIRECTION FINDING FOR DISTRIBUTED SIGNALS
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